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STANDARDS CRITERIA

Preamble: The Common Core State Standards define the rigorous skills and knowledge in English Language Arts and Mathematics that need to be effectively taught and learned for students to be ready to succeed academically in credit-bearing, college-entry courses and in workforce training programs. These standards have been developed to be:

- Fewer, clearer, and higher, to best drive effective policy and practice;
- Aligned with college and work expectations, so that all students are prepared for success upon graduating from high school;
- Inclusive of rigorous content and applications of knowledge through higher-order skills, so that all students are prepared for the 21st century;
- Internationally benchmarked, so that all students are prepared for succeeding in our global economy and society; and
- Research and evidence-based;

The goal of these standards is aspirational and meant to be forward looking yet based in evidence. The standards developed will set the stage for US education not just beyond next year, but for the next decade, and they must ensure ***all*** American students are prepared for the global economic workplace. Furthermore, the standards created will not lower the bar but raise it for all students; as such, we will seek to ensure all students are prepared for all entry-level, credit-bearing, academic college courses in English, mathematics, the sciences, the social sciences, and the humanities. The objective is for all students to enter these classes ready for success (defined for these purposes as a C or better).

Goal: The standards as a whole must be essential, rigorous, clear and specific, teachable and learnable, measureable, coherent, and internationally benchmarked.

Essential: Must be reasonable in scope in defining the knowledge and skills students should have to be ready to succeed in entry-level, credit-bearing, academic college courses and in workforce training programs.

Workforce training programs pertain to careers that:

1) Offer competitive, livable salaries above the poverty line
2) Offer opportunities for career advancement
3) Are in a growing or sustainable industry

College refers to: Two and four year postsecondary schools

Entry-level, credit-bearing, academic college courses refer to: English, mathematics, sciences, social sciences, humanities

Rigorous: Requires high-level cognitive demand by asking students to demonstrate deep conceptual understanding through the application of content knowledge and skills to new situations.

High-level cognitive demand refers to: reasoning, justification, synthesis, analysis, problem-solving.

Clear and Specific: Standards that provide sufficient guidance and clarity so that they are teachable, learnable, and measurable. The standards will also be clear and understandable to the general public.

Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive (the "what" not the "how"). The standards should maintain a relatively consistent level of grain size.

Teachable and learnable: Provide sufficient guidance for the design of curricula and instructional materials.

The standards must be reasonable in scope, instructionally manageable, and promote depth of understanding.

The standards will not prescribe *how* they are taught and learned but will allow teachers flexibility to teach and students to learn in various instructionally relevant contexts.

Measureable: Observable and verifiable and can be used to develop broader assessment frameworks.

Coherent: The standards should convey a unified vision of the big ideas and supporting concepts within a discipline and reflect a progression of learning that is meaningful and appropriate.

Grade-by-grade standards: The standards will have limited repetition across the grades or grade spans to help educators align instruction to the standards.

Internationally benchmarked: Be informed by the content, rigor, and organization of standards of high-performing countries so that all students are prepared for succeeding in our global economy and society.

Standards for Reading, Writing, and Communication

Table of Contents

The draft standards are based on evidence of what is required for college and career readiness, as well as benchmarking with other countries. To see a sample of the evidence supporting the core standards in reading, please go to the link below. Similar pages for writing and for speaking and listening are under development.

http://www.corestandards.net/readingmain.html

DRAFT
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Standards For Reading Informational and Literary Texts

Core Standards

To be college and career ready, students must:

1. Determine what the text says explicitly and use evidence within the text to infer what is implied by or follows logically from the text.
2. Support or question statements about the text by citing the text explicitly and accurately.
3. Assess the contributions that significant details as well as larger portions of the text make to the whole.
4. Summarize the ideas, events, or information in the text and determine the main ideas and themes.
5. Trace how events and ideas unfold in the text and explain how they relate to one another.
6. Analyze the traits, motivations, and thoughts of individuals in fiction and nonfiction based on how they are described, what they say and do, and how they interact.
7. Draw on context to determine what is meant by words and phrases, including figurative language.
8. Analyze how word choice shapes the meaning and tone of the text.
9. Analyze how the organizational structure advances the argument, explanation, or narrative.
10. Interpret data, graphics, and words in the text, and combine these elements of information to achieve comprehension.
11. Follow the reasoning that supports an argument or explanation and assess whether the evidence provided is relevant and sufficient.
12. Ascertain the origin and credibility of print and online sources when conducting research.
13. Analyze how two or more texts with different styles, perspectives, or arguments address similar topics or themes.
14. Apply knowledge and concepts drawn from texts to other texts, contexts, and circumstances.

Notes: The core standards are meant to apply to the different text types that students need to read for college and work. For example:

- "Trace how events and ideas unfold," applies to plot in literature and to a review of scientific procedures and explanations.
- "Analyze the traits, motivations, and thoughts of individuals," applies to studying characters in fiction and figures in historical texts.

Standards For Reading Informational and Literary Texts

Required Range and Contexts

To be college and career ready, students must read texts of sufficient complexity, quality, and range:

Complexity: A crucial factor in students' readiness for college and careers is their ability to read and comprehend complex text independently. Students must be able to handle high levels of text complexity with regard to the sophistication of the language and content as well as the subtlety of the themes and issues explored. In college and careers, students will need to extract knowledge and information from reference materials, technical manuals, literature, and other texts (print and online) that are characterized by demanding and context-dependent vocabulary, subtle relationships among ideas and characters, a nuanced rhetorical style and tone, and often elaborate structures or formats. These challenging texts require the reader's close attention and often demand rereading in order to be fully understood.

Quality: The literary and informational texts chosen for study should be rich in content. Since certain works are products of exceptional craft and thought, all students should have access to these especially strong models of thinking and writing. This includes texts that have broad resonance and are referred to and quoted often, such as influential political documents, foundational literary works, and seminal historical and scientific texts. At the same time, reading substantive contemporary fiction engages students in the world and culture around them, just as reading thoughtful contemporary works in science and other disciplines enables students to reflect on pertinent issues in these disciplines. Attentive and wide reading of high quality texts builds the background knowledge and vocabulary essential to college and career level reading comprehension.

Range: Students also must demonstrate their capacity to read a variety of literary and informational texts and read deeply within fields of study in order to gain the knowledge base they need for college and career readiness.

Literature: When reading literature, students must demonstrate their capacity to pay special attention to the choices authors make about words and structures. Many literary effects depend on the order in which events unfold and the specific details used to describe characters and actions. Since these same strategies—order and use of detail—are equally critical in understanding the most demanding informational texts, reading literature helps students comprehend what they read in science, history and other subjects.

Informational Text: Because the overwhelming majority of college and workplace reading is non-fiction, students need to hone their ability to acquire information from nonliterary texts in mathematics and the social and natural sciences. When reading informational text, students must become attuned to different formats in which ideas are presented to access the knowledge contained in these texts. In order to be college and career ready, students will need to encounter complex non-fiction in their English courses as well as when reading in history, the sciences and other disciplines.

Standards for Writing

Core Standards

To be college and career ready, students must:

1. Select and refine a topic or thesis that addresses the specific task and audience.
2. Sustain focus on a specific topic or argument through careful presentation of essential content.
3. Create a logical progression of ideas and use transitions effectively to convey the relationships among them.
4. Support and illustrate arguments and explanations with relevant details and examples.
5. Develop and maintain a style and tone appropriate to the purpose and audience.
6. Choose words and phrases to express ideas precisely and concisely.
7. Demonstrate command of the conventions of standard written English, including grammar, usage, and mechanics.
8. Represent and cite accurately the data, conclusions, and opinions of others.
9. Assess the quality of one's own writing and, when necessary, strengthen it through revision.

When **writing arguments**, students must also:

10. Establish a substantive claim, distinguishing it from alternate or opposing claims.
11. Link claims and evidence and ensure that the evidence is relevant and sufficient to support the claims.
12. Acknowledge competing arguments or information, defending or qualifying the initial claim as appropriate.

When **writing to inform or explain**, students must also:

13. Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information.
14. Convey complex information clearly and coherently to the audience through careful selection, organization, and presentation of the content.
15. Demonstrate understanding of the content by getting the key facts right, covering the essential points, and anticipating reader misconceptions.

Note: "The conventions of standard written English" encompass a range of commonly accepted language practices designed to make writing clear and widely understood. Correctness in writing is not an end in itself but rather a means to more effective communication. When formal writing contains errors in grammar, usage, and mechanics, its meaning is obscured, its message is too easily dismissed, and its author is often judged negatively. Proper sentence structure, correct verb formation, careful use of verb tense, clear subject-verb and pronoun-antecedent agreement, conventional usage, and appropriate punctuation that clarifies meaning are of particular importance to formal writing.

Standards for Writing

Required Range and Contexts

To be college and career ready, students must adapt their writing to the:

Purpose: Students must be able to accomplish two main purposes with their writing:

Make an Argument: The ability to frame and defend an argument is particularly important to students' readiness for college and careers. The goal of making an argument is to convince an audience of the rightness of the claims being made using logical reasoning and relevant evidence. In some cases, a student will make an argument to gain access to college or to a job, laying out their qualifications or experience. In college, a student might defend an interpretation of a work of literature or of history and, in the workplace, an employee might write to recommend a course of action. Students must frame the debate over a claim, presenting the evidence for the argument and acknowledging and addressing its limitations. This approach allows readers to test the veracity of the claims being made and the reasoning offered in their defense.

Inform or Explain: Writing to inform or explain requires students to integrate complex information from multiple sources in a lucid fashion, such as facts about a new technological application or a set of workplace procedures. To achieve coherence, students must illustrate the connections between ideas and events, such as cause and effect. Students also must organize their description or explanation in a manner appropriate to the context, responding to the specific needs of the reader by both covering the relevant ground and anticipating confusions that might arise. Writing is an opportunity for students to show what they know and share what they have seen, so it is essential that they check their facts and provide reliable information.

Audience: Students should write for a range of audiences and adapt their style and tone so that it is appropriate to the task and audience. Students must be able to take into consideration an audience's characteristics, such as its background knowledge, its interests, and its potential objections to an argument. Strong, effective writing can overcome or at least influence an audience's biases and address its limitations.

On-demand writing requirements of college and careers: Writers sometimes have the opportunity to take a piece of writing through multiple drafts, receiving feedback along the way, successively refining and polishing the text. Frequently, however, writers must produce high-quality text the first time and under a tight deadline, whether in response to a supervisor's request for information or to a prompt on an exam. To meet the special requirements of on-demand writing, writers must exhibit flexibility, concentration, and fluency.

Note on narrative writing

Narrative writing is an important component of making an argument and writing to inform or explain. Telling an interesting story effectively, faithfully describing the steps in a scientific process, or providing an accurate account of a historical incident requires skillfully using narrative techniques. Narrative writing requires that students present vivid, relevant details to situate events in a time and place and also craft a structure that lends a larger shape and significance to those details. As an easily grasped and widely used way to share information and ideas with others, narrative writing is a principal stepping-stone to writing forms directly relevant to college and career readiness.

College and Career Ready

Standards for Speaking and Listening

Core Standards

To be college and career ready, students must:

1. Present information and findings clearly and persuasively, selecting an appropriate format, organization, and register for the purpose and audience
2. Respond constructively to clarify points and to build on or challenge ideas.
3. Listen to complex information and understand what was said, identifying main ideas and supporting details.
4. Follow the progression of the speaker's message and evaluate the speaker's credibility and use of evidence.

Notes:

- Present information and findings clearly and persuasively: This includes conveying information concisely, taking into account audience background or prior knowledge of the selected topic, and ensuring that nonverbal cues such as gestures and eye contact contribute effectively to the delivery of the message.
- Register: This is the variety of language used in a particular setting. For example, a student should choose formal Standard English to deliver a presentation of research to an unfamiliar audience.
- Respond constructively: This can be accomplished by both a speaker and a listener. Responding constructively includes asking relevant questions, offering elaborations and answering questions, and using verbal and non-verbal cues to indicate or determine understanding, confusion, agreement, or disagreement.
- Evaluate the speaker's credibility and use of evidence: This includes distinguishing facts from opinions, determining bias and expertise, and assessing the speaker's supporting evidence.

Standards for Speaking and Listening

Required Range and Contexts

To be college and career ready, students must exhibit the Speaking and Listening Skills in the following contexts:

Formal and Informal:

Students are expected to exhibit the Speaking and Listening Skills in both formal and informal settings, adapting their language use accordingly. In particular, students should be able to use formal Standard English when called for in academic and workplace settings.

Group and One-to-One:

Students are expected to utilize the speaking and listening skills in both groups and one-to-one. The application of these skills may be different in varied settings. When communicating in a group and building on the ideas of others with group goals in mind, a student will have to respond constructively by taking turns, using non-verbal cues such as raising a hand. When communicating one-to-one, a student will be able to respond constructively in a more immediate manner such as by asking a question directly of the speaker.

Applications of the Core

Research

Note: This section draws on the Reading, Writing, and Speaking and Listening standards to demonstrate important applications of the core to college and career readiness.

The ability to conduct research independently, accurately, and effectively plays a fundamental role in college and the workplace. Research skills are critical tools for acquiring, extending, and sharing knowledge in academic and workplace settings, and students must be able to determine when and how to conduct and document research.

Research as described here is not limited to the formal, extended research paper; rather, these skills encompass a flexible yet systematic approach to resolving questions and investigating issues through the careful collection, analysis, synthesis, and presentation of information from print and digital sources. These research skills equip students with the tools to engage in sustained inquiry as well as tackle short, focused research projects that typify many research assignments in college and the workplace. Research in the digital age offers new possibilities but also new or heightened challenges. For one, the explosion of information available electronically puts a premium on students being able to determine the origin and credibility of their sources.

To be college and career ready, students must engage in research and present their findings in writing and orally, in print and online. While the skills represented in many of the standards from Reading, Writing, and Speaking and Listening could be called on when performing research, the following encapsulate the core standards for this application:

Reading:

- Summarize the ideas, events, or information in the text and determine the main ideas and themes. (4)
- Interpret data, graphics, and words in the text, and combine these elements of information to achieve comprehension. (10)
- Follow the reasoning that supports an argument or explanation and assess whether the evidence provided is relevant and sufficient. (11)
- Ascertain the origin and credibility of print and online sources when conducting research. (12)
- Apply knowledge and concepts drawn from texts to other texts, contexts, and circumstances. (13)

Writing:

- Select and refine a topic or thesis that addresses the specific task and audience. (1)
- Represent and cite accurately the data, conclusions, and opinions of others. (6)
- Establish a substantive claim, distinguishing it from alternate or opposing claims. (10)
- Link claims and evidence and ensure that the evidence is relevant and sufficient to support the claims. (11)
- Acknowledge competing information or arguments, defending or qualifying the initial claim as appropriate. (12)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to develop an accurate picture of that information. (13)
- Convey complex information clearly and coherently to the audience through careful selection, organization, and presentation of the content. (14)
- Demonstrate understanding of the content by getting the key facts right, covering the essential points, and anticipating reader misconceptions. (15)

Speaking and Listening:

- Present information and findings clearly and persuasively, selecting an appropriate format, organization, and register for the purpose and audience. (1)

Applications of the Core

Media

Media skills play an increasingly important role in the gathering and sharing of ideas and information. At the core of media mastery are the same fundamental capacities as are required "offline" in traditional print forms: an ability to produce clear communications and an ability to access, understand, and evaluate complex materials and messages.

Media mastery also calls upon some skills unique to the online environment, ranging from being able to conduct digital-based research to exchanging and debating ideas in online discussions to interacting with new text forms. In the electronic world, reading and writing are closely intertwined, which affects both the processing of information as well as its production. Students should be able to create, collaborate on, and distribute media communications and must learn both to read closely and critically and to contribute effectively online through different media forms, such as blogs, wikis, and social networks.

While the skills represented in many of the standards from Reading, Writing, and Speaking and Listening could be called on in the interpretation and production of media, the following encapsulate the core standards for this application:

Reading:

- Summarize the ideas, events, or information in the text and determine the main ideas and themes. (4)
- Interpret data, graphics, and words in the text, and combine these elements of information to achieve comprehension. (10)
- Ascertain the origin and credibility of print and online sources when conducting research. (12)
- Analyze how two or more texts with different styles, perspectives, or arguments address similar topics or themes. (13)

Writing:

- Represent and cite accurately the data, conclusions, and opinions of others. (6)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to develop an accurate picture of that information. (13)
- Convey complex information clearly and coherently to the audience through careful selection, organization, and presentation of the content. (14)
- Demonstrate understanding of the content by getting the key facts right, covering the essential points, and anticipating reader misconceptions. (15)

Speaking and Listening

- Present information and findings clearly and persuasively, selecting an appropriate format, organization, and register for the purpose and audience. (1)
- Listen to complex information and understand what was said, identifying main ideas and supporting details. (3)
- Follow the progression of the speaker's message and evaluate the speaker's credibility and use of evidence. (4)

Reading Illustrative Texts at the Required Level of Complexity

Significance and Measurement of Text Complexity

Why Samples of Complex Texts?

Studies show that one concrete measure of readiness for college and careers is students' ability to read and comprehend complex text independently. Many students who do not encounter sufficiently challenging texts in high school struggle upon entering college. In the twenty-first century, students may change jobs often; they must therefore be able to read a range of complex texts to be ready for an ever-evolving workplace. While no sampling can do justice to the numerous ways in which different authors craft engaging and complex prose, the four selections below exemplify the kinds of passages students need to grapple with in high school to be ready to meet the challenges of college classrooms and workplaces.

- The first selection, from the Declaration of Independence, illustrates the kind of primary source materials students should be able to confront on their own in high school.
- The second passage, from Katherine Mansfield's short story, "Miss Brill," appears on several international reading lists and in many U.S. high school curricula.
- The third excerpt, by Sylvia Mader, comes from an entry-level college science text.
- The fourth text, taken from one of ACT's WorkKeys® assessments, represents one important type of real-world reading challenge: the business memo.

How Has Complexity Been Measured?

In addition to surveys of required reading in twelfth grade and the first year of college and consultations with experts in text complexity, two leading measurement systems were used to help make the selections below. While other measures of readability and text complexity have value, the two described below helped guide this initial work and confirmed that these four texts are suitable exemplars of the types of complex texts that students need to master to be ready for college and careers.

The first system—a methodology described by Jeanne Chall and her coauthors in *The Qualitative Assessment of Text Difficulty*—employs trained raters to measure the sophistication of vocabulary, density of ideas, and syntactic complexity in a text as well as the general and subject-specific knowledge and the level of reasoning required for understanding it. The second system, Coh-Metrix, incorporates into its computer-based analysis more than sixty specific indices of syntax, semantics, readability, and cohesion to assess text complexity. Central to its assessment are measures of text cohesiveness, that is, the degree to which the text uses explicit markers to link ideas. By analyzing the degree to which those links are missing in a text—and therefore the degree to which a reader must make inferences to connect ideas—this measure gauges a key factor in the comprehension demand of a text.

Sample Text #1

from The Declaration of Independence

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable rights, that among these are life, liberty and the pursuit of happiness. That to secure these rights, governments are instituted among men, deriving their just powers from the consent of the governed. That whenever any form of government becomes destructive to these ends, it is the right of the people to alter or to abolish it, and to institute new government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their safety and happiness. Prudence, indeed, will dictate that governments long established should not be changed for light and transient causes; and accordingly all experience hath shown that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same object evinces a design to reduce them under absolute despotism, it is their right, it is their duty, to throw off such government, and to provide new guards for their future security. --Such has been the patient sufferance of these colonies; and such is now the necessity which constrains them to alter their former systems of government. The history of the present King of Great Britain is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute tyranny over these states. To prove this, let facts be submitted to a candid world.

Sample Text #2

from "Miss Brill," by Katherine Mansfield

There were a number of people out this afternoon, far more than last Sunday. And the band sounded louder and gayer. That was because the Season had begun. For although the band played all the year round on Sundays, out of season it was never the same. It was like some one playing with only the family to listen; it didn't care how it played if there weren't any strangers present. Wasn't the conductor wearing a new coat, too? She was sure it was new. He scraped with his foot and flapped his arms like a rooster about to crow, and the bandsmen sitting in the green rotunda blew out their cheeks and glared at the music. Now there came a little "flutey" bit—very pretty! —a little chain of bright drops. She was sure it would be repeated. It was; she lifted her head and smiled.

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The band had been having a rest. Now they started again. And what they played was warm, sunny, yet there was just a faint chill—a something, what was it? —not sadness—no, not sadness—a something that made you want to sing. The tune lifted, lifted, the light shone; and it seemed to Miss Brill that in another moment all of them, all the whole company, would begin singing. The young ones, the laughing ones who were moving together, they would begin, and the men's voices, very resolute and brave, would join them. And then she too, she too, and the others on the benches—they would come in with a kind of accompaniment—something low, that scarcely rose or fell, something so beautiful—moving— And Miss Brill's eyes filled with tears and she looked smiling at all the other members of the company. Yes, we understand, we understand, she thought—though what they understood she didn't know.

Sample Text #3

***from* Inquiry into Life, by Sylvia S. Mader, 12th edition. (McGraw Hill)**

A **covalent bond** results when two atoms share electrons in such a way that each atom has an octet of electrons in the outer shell. In a hydrogen atom, the outer shell is complete when it contains two electrons. If hydrogen is in the presence of a strong electron acceptor, it gives up its electron to become a hydrogen ion (H^+). But if this is not possible, hydrogen can share with another atom and thereby have a completed outer shell. For example, one hydrogen atom will share with another hydrogen atom. Their two orbitals overlap, and the electrons are shared between them. Because they share the electron pair, each atom has a completed outer shell.

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The passage of salt (NaCl) across a plasma membrane is of primary importance to most cells. The chloride ion (Cl^-) usually crosses the plasma membrane because it is attracted by positively charged sodium ions (Na^+). First sodium ions are pumped across a membrane, and then chloride ions simply diffuse through channels that allow their passage.

As noted in Figure 4.2*a*, the genetic disorder cystic fibrosis results from a faulty chloride channel. Ordinarily, after chloride ions have passed though the membrane, sodium ions (Na^+) and water follow. In cystic fibrosis, Cl^- transport is reduced, and so is the flow of Na^+ and water.

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Once a neurotransmitter has been released into a synaptic cleft and has initiated a response, it is removed from the cleft. In some synapses, the postsynaptic membrane contains enzymes that rapidly inactivate the neurotransmitter. For example, the enzyme **acetylcholinesterase (AChE)** breaks down acetylcholine. In other synapses, the presynaptic membrane rapidly reabsorbs the neurotransmitter, possibly for repackaging in synaptic vesicles or for molecular breakdown. The short existence of neurotransmitters at a synapse prevents continuous stimulation (or inhibition) of postsynaptic membranes.

Sample Text #4

Sample Business Memo

(ACT WorkKeys Reading for Information Test, Level 6 Sample Passage)

To permit our employees to communicate directly with one another as well as with vendors and customers, Molten Metals, Inc. provides a network of e-mail accounts. Access to e-mail is at the sole discretion of Molten Metals, Inc., and we will determine who is to be so empowered. Under President Duarte's leadership, all messages sent and received (even those intended as personal) are treated as business messages. Molten Metals, Inc. has the capability to and reserves the right to access, review, copy, and delete any messages sent, received, or stored on the company e-mail server. Molten Metals, Inc. will disclose these messages to any party (inside or outside the company) it deems appropriate. Employees should treat this server as a constantly reviewed, shared file stored in the system.

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Due to the reduced human effort required to redistribute electronic information, a greater degree of caution must be exercised by employees transmitting MM, Inc. confidential information using company e-mail accounts. Confidential information belonging to MM, Inc. is important to our independence and should never be transmitted or forwarded to persons or companies not authorized to receive that information. Likewise, it should not be sent or forwarded to other employees inside the company who do not need to know that information.

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MM, Inc. strongly discourages the storage of large numbers of e-mail messages for a number of reasons. First, because e-mail messages frequently contain company confidential information, it is good to limit the number of such messages to protect the company's information. Second, retention of messages fills up large amounts of storage space on the e-mail server and personal hard disks, and can slow down the performance of both the network and individual personal computers. Finally, in the event that the company needs to search the network server, backup tapes, or individual hard disks for genuinely important documents, the fewer documents it has to search through, the more economical the search will be. Therefore, employees are to delete as soon as possible any email messages they send or receive.

College and Career Readiness Standards for Mathematics

Draft for Review and Comment

July 16, 2009

Contents

Introduction and Overview of the Organization

Ten *Mathematical Principles* form the backbone of these standards. Each principle is accompanied by an explanation that describes the coherent view students are expected to have of a specific area of mathematics. With this coherent view, students will be better able to learn more mathematics and use the mathematics they know. The principles pull together topics previously studied and target topics yet to be learned in post-secondary programs.

Each principle consists of a statement of a Coherent Understanding of the principle, together with Core Concepts, Core Skills, and Explanatory Problems that exemplify and delimit the range of tasks students should be able to do.

These standards, like vectors, specify direction and distance for students to be ready for college and careers:

1. Direction—The Coherent Understanding

   The Coherent Understandings attempt to communicate the mathematical coherence of the knowledge students should take into college and careers. They are intended to tell teachers, 'This is how your students should see the mathematics in this area in order to aim them towards mastering it.'

2. Distance—The Concepts, Skills and Explanatory Problems

   Collectively, these statements and sets of problems define and clarify the level of expertise students should reach if they are to be prepared for success in college and career. They are

   a. statements of concepts students must know and actions students must be able to take using the mathematics; and

   b. examples of the problems and other assignments they must be able to complete.

In addition to the *Mathematical Principles*, the standards also contain a set of *Mathematical Practices* that are key to using mathematics in the workplace, in further education and in a 21st Century democracy. Students who care about being precise, who look for hidden structure and note regularity in repeated reasoning, who make sense of complex problems and persevere in solving them, who construct viable arguments and use technology intelligently are more likely to be able to apply the knowledge they have attained in school to new situations. These mathematical practices are described and tied to examples.

Taken together, the explanations of the mathematical principles, the associated concepts and skills and the mathematical practices form the College and Career Readiness Standards for Mathematics.

Overview of the Mathematical Principles

Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations and are the foundation of algebra.

Expressions. Expressions use symbols and efficient notational conventions about order of operations, fractions and exponents to express verbal descriptions of computations in a compact form.

Equations. An equation is a statement that two expressions are equal, which may result from expressing the same quantity in two different ways, or from asking when two different quantities have the same value. Solving an equation means finding the values of the variables in it that make it true.

Functions. Functions describe the dependence of one quantity on another. For example, the return on an investment is a function of the interest rate. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Quantity. A quantity is an attribute of an object or phenomenon that can be measured using numbers. Specifying a quantity pairs a number with a unit of measure, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

Shape. Shapes, their attributes, and the relations among them can be analyzed and generalized using the deductive method first developed by Euclid, generating a rich body of theorems from a few axioms.

Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Probability. Probability assesses the likelihood of an event. It allows for the quantification of uncertainty, describing the degree of certainty that an event will happen as a number from 0 through 1.

Statistics. We often base decisions or predictions on data. The decisions or predictions would be easy to make if the data always sent a clear signal, but the signal is usually obscured by noise. Statistical analysis aims to account for both the signal and the noise, allowing decisions to be as well informed as possible.

College and Career Readiness Standards for Mathematics

Mathematical Practices

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically. The mathematical practices described below bind together the five strands of mathematical proficiency: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition.[a]

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls.[b] They learn that effort counts in mathematical achievement.[c] These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures.[d]

1. They care about being precise.

Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies based on the assumptions made. They clarify definitions. They state the meaning of the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. They would never say "let v be speed and let t be elapsed time" but rather "let v be the speed in meters per second and let t be the elapsed time in seconds." They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits are meaningless.

2. They construct viable arguments.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others.

3. They make sense of complex problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for the entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

4. They look for structure.

Mathematically proficient students look closely to discern a pattern or stand back to get an overview or shift their perspective, and they transfer fluently between these points of view. For example, in $x^2 + 5x + 6$ they can see the 5 as $2 + 3$ and the 6 as 2×3 They recognize the significance of an existing line in a geometric figure or add an auxiliary line to make the solution of a problem clear. They also can step back and see complicated things, such as some algebraic expressions, as single objects that they can manipulate. For example, they might determine that the value of $5 - 3(x - y)^2$ is at most 5 because $(x - y)^2$ is non-negative.[d]

5. They look for and express regularity in repeated reasoning.

Mathematically proficient students pay attention to repeated calculations as they are carrying them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, they might abstract an equation of the line of the form $\frac{y-2}{x-1} = 3$. By noticing the telescoping in the expansions of $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$, they might derive the general formula for the sum of a geometric series. As they work through the solution to a problem, they maintain oversight over the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.[d]

6. They make strategic decisions about the use of technological tools.

Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, graphing calculators, spreadsheets, dynamic geometry or statistical software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding, attention to levels of precision and estimation to provide realistic levels of approximation and to detect possible errors.

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a) The term *proficiency* is used here as it was defined in the 2001 National Research Council report *Adding it up: Helping children learn mathematics*. The term was used in the same way by the National Mathematics Advisory Panel (2008).

b) Singapore standards

c) National Mathematics Advisory Panel (2008)

d) Cuoco, A. , Goldenberg, E. P., and Mark, J. (1996). *Journal of Mathematical Behavior*, 15 (4), 375-402; *Focus in High School Mathematics*. Reston, VA: NCTM, in press.

Number

Core Concepts · Students understand that:

A Standard algorithms are based on place value and the rules of arithmetic.

B Fractions represent numbers. Equivalent fractions have the same value.

C All real numbers can be located on the number line.

A Coherent Understanding of Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations and are the foundation of algebra.

The place value system bundles units into 10s, then 10s into 100s, and so on, providing a method for naming large numbers. Subdividing in a similar way extends this to the decimal system for naming all real numbers and locating them on a number line. This system is the basis for efficient algorithms. Numbers represented as fractions, such as rational numbers, can also be located on the number line by seeing them as numbers expressed in different units (for example, 3/5 is three fifths).

Operations with fractions depend on applying the rules of arithmetic:

- Numbers can be added in any order with any grouping and multiplied in any order with any grouping.
- Multiplication by 1 and addition of 0 leave numbers unchanged.
- All numbers have additive inverses, and all numbers except zero have multiplicative inverses.
- Multiplication distributes over addition.

Mental computation strategies are opportunistic uses of these rules, which, for example, allow one to compute the product 5×177×2 at a glance, obtaining 1770 instantly rather than methodically working from left to right.

Sometimes an estimate is more appropriate than an exact value. For example, it might be more useful to give the length of a board approximately as 1 ft $4\frac{3}{4}$ in, rather than exactly as $\sqrt{2}$ ft; an estimate of how long a light bulb lasts helps in determining the number of light bulbs to buy. In addition, estimation and approximation are useful in checking calculations.

Connections to Expression, Equations and Functions. The rules of arithmetic govern the manipulations of expressions and functions and, along with the properties of equality, provide a foundation for solving equations.

Core Skills · Students can and do:

1 Use standard algorithms with procedural fluency.*

2 Use mental strategies and technology with strategic competence.**

3 Compare numbers and make sense of their magnitude.

Include positive and negative numbers expressed as fractions, decimals, powers and roots. Limit to square and cube roots. Include very large and very small numbers.

4 Solve multi-step problems involving fractions and percentages.

Include situations such as simple interest, tax, markups/markdowns, gratuities and commissions, fees, percent increase or decrease, percent error, expressing rent as a percentage of take-home pay, and so on. Students should also be able to solve problems of the three basic forms: 25 percent of 12 is what? 3 is what percent of 12? and 3 is 25 percent of what? and understand how these three problems are related.

5 Use estimation to solve problems and detect errors.

6 Give answers to an appropriate level of precision.

* The term procedural fluency as used in this document has the same meaning as in the National Research Council report *Adding it up: Helping children learn mathematics*. Specifically, "*Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121).

** The term strategic competence as used in this document has the same meaning as in the National Research Council report *Adding it up: Helping children learn mathematics*. Specifically, "*Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them" (p. 124).

Expressions

Core Concepts · Students understand that:

A Expressions represent computations with symbols standing for numbers.

B Complex expressions are made up of simpler expressions.

C Rewriting expressions serves a purpose in solving problems.

A Coherent Understanding of Expressions. Expressions use symbols and efficient notational conventions about order of operations, fractions and exponents to express verbal descriptions of computations in a compact form.

For example, $p + 0.05p$ expresses the addition of a 5% tax to a price p. Reading an expression with comprehension involves analysis of its underlying structure, which may suggest a different but equivalent way of writing it that exhibits some different aspect of its meaning. For example, rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying by a constant factor.

Heuristic mnemonic devices are not a substitute for procedural fluency, which depends on understanding the basis of manipulations in the rules of arithmetic and the conventions of algebraic notation. For example, factoring, expanding, collecting like terms, the rules for interpreting minus signs next to parenthetical sums, and adding fractions with a common denominator are all instances of the distributive law; the interpretation we give to negative and rational exponents is based on the extension of the exponent laws for positive integers to negative and rational exponents. When simple expressions within more complex expressions are treated as single quantities, or chunks, the underlying structure of the larger expression may be more evident.

Connections to Equations and Functions. Setting expressions equal to each other leads to equations. Expressions can define functions, with equivalent expressions defining the same function.

Core Skills · Students can and do:

1 See structure in expressions and manipulate simple expressions with procedural fluency.

See Explanatory Problems.

2 Write an expression to represent a quantity in a problem.

3 Interpret an expression and its parts in terms of the quantity it represents.

See Explanatory Problems.

Equations

Core Concepts · Students understand that:

A An equation is a statement that two expressions are equal.

B Solving is a process of algebraic manipulation guided by logical reasoning.

C Completing the square leads to a formula for solving quadratic equations.

D Equations not solvable in one number system might be solvable in a larger system.

A Coherent Understanding of Equations. An equation is a statement that two expressions are equal, which may result from expressing the same quantity in two different ways, or from asking when two different quantities have the same value. Solving an equation means finding the values of the variables in it that make it true.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs, which can be graphed in the plane. Equations can be combined into systems to be solved simultaneously.

An equation can be solved by successively transforming it into one or more simpler equations. The process is governed by deductions based on the properties of equality. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Some equations have no solutions in a given number system, stimulating the formation of expanded number systems (integers, rational numbers, real numbers and complex numbers). Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

A formula expressing a general relationship among several variables is a type of equation, and the same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \left(\frac{b_1+b_2}{2}\right)h$, can be solved for h using the same deductive steps.

Like equations, inequalities can involve one or more variables and can be solved in much the same way. Many, but not all, of the properties of equality extend to the solution of inequalities.

Connections to Functions, Coordinates, and Modeling. Equations in two variables can define functions, and questions about when two functions have the same value lead to equations. Graphing the functions allows for the approximate solution of equations. Equations of lines are addressed under Coordinates, and converting verbal descriptions to equations is addressed under Modeling.

Core Skills · Students can and do:

1 Understand a word problem and restate it as an equation.

Extend to inequalities and systems.

2 Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality.

Solve linear equations with procedural fluency. For quadratic equations, include solution by inspection, by factoring, or by using the quadratic formula. *See Explanatory Problems.*

3 Rearrange formulas to isolate a quantity of interest.

Exclude cases that require extraction of roots or inverse functions.*

4 Solve systems of equations.

Focus on pairs of simultaneous linear equations in two variables. Include algebraic techniques, graphical techniques and solving by inspection.

5 Solve linear inequalities in one variable and graph the solution set on a number line.

6 Graph the solution set of a linear inequality in two variables on the coordinate plane.

*Exclusions of this sort are modeled after Singapore's standards, which contains similar exclusions and limitations to help define the desired level of complexity.

Functions

Core Concepts · Students understand that:

A A function describes the dependence of one quantity on another.

B The graph of a function f is a set of ordered pairs $(x, f(x))$ in the coordinate plane.

C Common functions occur in parametric families where each member describes a similar type of dependence.

A Coherent Understanding of Functions. Functions describe the dependence of one quantity on another. For example, the return on an investment is a function of the interest rate. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Functions in school mathematics are often presented by an algebraic rule. For example, the time in hours it takes for a plane to fly 1000 miles is a function of the plane's speed in miles per hour; the rule $T(s) = 1000/s$ expresses this dependence algebraically and is said to define a function, whose name is T. The graph of a function is a useful way of visualizing the dependency it models, and manipulating the expression for a function can throw light on its properties. Sometimes functions are defined by a recursive process which can be modeled effectively using a spreadsheet or other technology.

Two important families of functions are characterized by laws of growth: linear functions grow at a constant rate, and exponential functions grow at a constant percent rate. Linear functions with an initial value of zero describe proportional relationships.

Connections to Expressions, Equations, Modeling and Coordinates. Functions may be defined by expressions. The graph of a function f is the same as the solution set of the equation $y = f(x)$. Questions about when two functions have the same value lead to equations, whose solutions can be visualized from the intersection of the graphs. Since functions express relationships between quantities, they are frequently used in modeling.

Core Skills · Students can and do:

1 Recognize proportional relationships and solve problems involving rates and ratios.

2 Describe the qualitative behavior of common types of functions using expressions, graphs and tables.

Use graphs and tables to identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; and periodicity. Explore the effects of parameter changes (including shifts and stretches) on the graphs of these functions using technology. Include linear, power, quadratic, polynomial, simple rational, exponential, logarithmic, trigonometric, absolute value and step functions. *See Explanatory Problems.*

3 Analyze functions using symbolic manipulation.

Include slope-intercept and point-slope form of linear functions; factored form to find horizontal intercepts; vertex form of quadratic functions to find maximums and minimums; and manipulations as described under Expressions. *See Explanatory Problems.*

4 Use the families of linear and exponential functions to solve problems.

For linear functions $f(x) = mx + b$, understand b as the intercept or initial value and m as the slope or rate of change. For exponential functions $f(x) = a{\cdot}b^x$, understand a as the intercept or initial value and b as the growth factor. *See Explanatory Problems.*

5 Find and interpret rates of change.

Compute the rate of change of a linear function and make qualitative observations about the rates of change of nonlinear functions.

Quantity

Core Concepts · Students understand that:

A The value of a quantity is not specified unless the units are named or understood from the context.

B Quantities can be added and subtracted only when they are of the same general kind (lengths, areas, speeds, etc.).

C Quantities can be multiplied or divided to create new types of quantities, called derived quantities.

A Coherent Understanding of Quantity. A quantity is an attribute of an object or phenomenon that can be measured using numbers. Specifying a quantity pairs a number with a unit of measure, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

For example, the length of a football field and the speed of light are both quantities. If we choose units of miles per second, then the speed of light has the value 186,000 miles per second. But the speed of light need not be expressed in second per hour; it may be expressed in meters per second or any other unit of speed. A speed of 186,000 miles per second is the same as a speed of meters per second. "Bare" numerical values such as 186,000 and do not describe quantities unless they are paired with units.

Speed (distance divided by time), rectangular area (length multiplied by length), density (mass divided by volume), and population density (number of people divided by area) are examples of derived quantities, obtained by multiplying or dividing quantities.

It can make sense to add two quantities, such as when a child 51 inches tall grows 3 inches to become 54 inches tall. To be added or subtracted, quantities must be expressed in the same units, but even then it does not always make sense to add them. If a wooded park with 300 trees per acre is next to a field with 30 trees per acre, they do not have 330 trees per acre. Converting quantities to have the same units is like converting fractions to have a common denominator before adding or subtracting.

Doing algebra with units in a calculation reveals the units of the answer, and can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed.

Connections to Number, Expressions, Equations, Functions and Modeling. Operations described under Number and Expressions govern the operations one performs on quantities, including the units involved. Quantity is an integral part of any application of mathematics, and has connections to solving problems using equations, functions and modeling.

Core Skills · Students can and do:

1 Use units consistently in describing real-life measures, including in data displays and graphs.

2 Know when and how to convert units in computations.

Include the addition and subtraction of quantities of the same general kind expressed in different units; averaging data given in mixed units; converting units for derived quantities such as density and speed.

3 Use and interpret derived quantities and units correctly in algebraic formulas.

4 Use units as a way to understand problems and to guide the solution of multi-step problems.

Include examples such as acceleration; currency conversions; people-hours; social science measures, such as deaths per 100,000; and general rate, such as points per game. *See Explanatory Problems.*

Modeling

Core Concepts · Students understand that:

A Models abstract key features from situations to help us solve problems.

B Models can be useful even if their assumptions are oversimplified.

A Coherent Understanding of Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

A model can be very simple, such as a geometric shape to describe a physical object like a coin. Even so simple a model involves making choices. It is up to us whether to model the solid nature of the coin with a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. For some purposes, we might even choose to adjust the right circular cylinder to model more closely the way the coin deviates from the cylinder.

In any given situation, the model we devise depends on a number of factors: How exact an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models we can create and analyze is constrained as well by the limitations of our mathematical and technical skills. For example, modeling a physical object, a delivery route, a production schedule, or a comparison of loan amortizations each requires different sets of tools. Networks, spreadsheets and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

The basic modeling cycle is one of (1) apprehending the important features of a situation, (2) creating a mathematical model that describes the situation, (3) analyzing and performing the mathematics needed to draw conclusions from the model, and (4) interpreting the results of the mathematics in terms of the original situation.

Connections to Quantity, Equations, Functions, Shape and Statistics. Modeling makes use of shape, data and algebra to represent real-world quantities and situations. In this way the Modeling Principle relies on concepts of quantity, equations, functions, shape and statistics.

Core Skills · Students can and do:

1 Model numerical situations.

Include readily applying the four basic operations in combination to solve multi-step quantitative problems with dimensioned quantities; making estimates to introduce numbers into a situation and get a problem started; recognizing proportional or near-proportional relationships and analyzing them using characteristic rates and ratios.

2 Model physical objects with geometric shapes.

Include common objects that can reasonably be idealized as two- and three-dimensional geometric shapes. Identify the ways in which the actual shape varies from the idealized geometric model.

3 Model situations with equations, inequalities and functions.

Include situations well described by a linear inequality in two variables or a system of linear inequalities that define a region in the plane; situations well described by linear, quadratic or exponential equations or functions; and situations that can be well described by inverse variation.

4 Model situations with common functions.

Include identifying a family that models a problem and identify a particular function of that family adjusting parameters. Understand the recursive nature of situations modeled by linear and exponential functions.

5 Model data with statistics.

Include replacing a distribution of values with a measure of its central tendency; modeling a bivariate relationship using a trend line or a linear regression line.

6 Compare models for a situation.

Include recognizing that there can be many models that relate to a situation, that they can capture different aspects of the situation, that they can be simpler or more complex, and that they can have a better or worse fit to the situation and the questions being asked.

7 Interpret the results of applying the model in the context of the situation.

Include realizing that models do not always fit exactly and so there can be error; identifying simple sources of error and being careful not to over-interpret models.

Shape

Core Concepts · Students understand that:

A Shapes, their attributes, and their measurements can be analyzed deductively.

B Right triangles and the Pythagorean theorem are focal points in geometry with practical and theoretical importance.

C Congruent shapes can be superimposed through rigid transformations.

D Proportionality governs the relationship between measurements of similar shapes.

A Coherent Understanding of Shape. Shapes, their attributes, and the relations among them can be analyzed and generalized using the deductive method first developed by Euclid, generating a rich body of theorems from a few axioms.

The analysis of an object rests on recognition of the points, lines and surfaces that define its shape: a circle is a set of points in a plane equidistant from a fixed point; a cube is a figure composed of six identical square regions in a particular three-dimensional arrangement. Precise definitions support an understanding of the ideal, allowing application to the real world where geometric modeling, measurement, and spatial reasoning offer ways to interpret and describe physical environments.

We can also analyze shapes, and the relations of congruence and symmetry, through transformations such as translations, reflections, and rotations. For example, the line of reflective symmetry in an isosceles triangle assures that its base angles are equal.

The study of similar right triangles supports the definition of sine, cosine and tangent for acute angles, and the Pythagorean theorem is a key link between shape, measurement, and coordinates. Knowledge about triangles and measurement can be applied in practical problems, such as estimating the amount of wood needed to frame a sloping roof.

Connections to Coordinates and Functions. The Pythagorean theorem provides an important bridge between shape and distance in the coordinate plane. Parameter changes in families of functions can be interpreted as transformations applied to their graphs.

Core Skills · Students can and do:

1 Use geometric properties to solve multi-step problems involving shapes.

Include: measures of angles of a triangle sum to 180°; measures of vertical, alternate interior and corresponding angles are equal; measures of supplemental angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are equidistant from the segment's endpoints; and the radius of a circle is perpendicular to the tangent at the point of intersection of the circle and radius. *See Explanatory Problems.*

2 Prove theorems, test conjectures and identify logical errors.

Include theorems about angles, parallel and perpendicular lines, similarity and congruence of triangles.

3 Solve problems involving measurements.

Include measurement (length, angle measure, area, surface area, and volume) of a variety of figures and shapes in two- and three-dimensions. Compute measurements using formulas and by decomposing complex shapes into simpler ones. *See Explanatory Problems.*

4 Construct shapes from a specification of their properties using a variety of tools.

Include classical construction techniques and construction techniques supported by modern technologies.

5 Solve problems about similar triangles and scale drawings.

Include computing actual lengths, areas and volumes from a scale drawing and reproducing a scale drawing at a different scale.

6 Apply properties of right triangles and right triangle trigonometry to solve problems.

Include applying sine, cosine and tangent to determine lengths and angle measures of a right triangle, the Pythagorean theorem and properties of special right triangles. Use right triangles and their properties to solve real-world problems. Limit angle measures to degrees. *See Explanatory Problems.*

7 Create and interpret two-dimensional representations of three-dimensional objects.

Include schematics, perspective drawings and multiple views.

Coordinates

Core Concepts · Students understand that:

A Locations in space can be described using numbers called coordinates.

B Coordinates serve as tools for blending algebra with geometry and allow methods from one domain to be used to solve problems in the other.

C The set of solutions to an equation in two variables is a line or curve in the coordinate plane and the solutions to systems of equations in two variables correspond to intersections of lines or curves.

D Equations in different families graph as different sorts of curves—such as straight lines, parabolas, circles.

A Coherent Understanding of Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be cast as equations, making algebraic manipulation into a tool for geometric proof and understanding.

Coordinate geometry is a rich field for exploration. How does a geometric transformation such as a translation or reflection affect the coordinates of points? What features does the graph have for a rational function whose denominator can be zero? How is the geometric definition of a circle reflected in its equation?

Coordinates can also be applied to scale maps and provide a language for talking about direction and bearing. Adding a third perpendicular axis associates three numbers with locations in three dimensions and extends the use of algebraic techniques to problems involving the three-dimensional world we live in.

Connections to Shape, Quantity, Equations and Functions. Coordinates can be used to reason about shapes. In applications, coordinates often have dimensions and units (such as lengths and bushels). A one-variable equation of the form $f(x) = g(x)$ may be solved in the coordinate plane by finding intersections of the curves $y = f(x)$ and $y = g(x)$.

Core Skills · Students can and do:

1 Translate fluently between lines in the coordinate plane and their equations.

Include predicting visual features of lines by inspection of their equations, determining the equation of the line through two given points, and determining the equation of the line with a given slope passing through a given point.

2 Identify the correspondence between parameters in common families of equations and the shape of their graphs.

Include common families of equations—the graphs of $Ax + By = C$, $y = mx + b$ and $x = a$ are straight lines; the graphs of $y = a(x - h)^2 + k$ and $y = Ax^2 + Bx + C$ are parabolas; and the graph of $(x - h)^2 + (y - k)^2 = r^2$ is a circle.

3 Use coordinates to solve geometric problems.

Include proving simple theorems algebraically, using coordinates to compute perimeters and areas for triangles and rectangles, finding midpoints of line segments, finding distances between pairs of points and determining the parallelism or perpendicularity of lines. *See Explanatory Problems.*

Probability

Core Concepts · Students understand that:

A Probability expresses a rational degree of certainty with a number from 0 to 1 where probability of 1 means that an event is certain.

B When there are *n* equally likely outcomes the probability of any one of them occurring is $\frac{1}{n}$ and the probability of any combination of outcomes can be computed using the laws of probability.

C Probability is an important consideration in rational decision-making.

A Coherent Understanding of Probability. Probability assesses the likelihood of an event. It allows for the quantification of uncertainty, describing the degree of certainty that an event will happen as a number from 0 through 1.

In some situations, such as flipping a coin, rolling a number cube or drawing a card, where no bias exists for or against any particular outcome, it is reasonable to assume that the possible outcomes are all equally likely. From this assumption the laws of probability give the probability for each possible number of heads, sixes or aces after a given number of trials. Generally speaking, if you know the probabilities of some simple events you can use the laws of probability to deduce probabilities of combinations of them.

An important method in such calculations is systematically counting all the possibilities in a situation. Systematic counting often involves arranging the objects to be counted in such a way that the problem of counting reduces to a smaller problem of the same kind.

In some situations it is not known whether an event has been influenced by outside factors. If we question whether a number cube is fair, we can compare the results we get by rolling it to the frequencies predicted by the mathematical model. It is this application of probability that underpins drawing valid conclusions from sampling or experimental data. For example, if the experimental population given a drug is categorized 20 different ways, a manufacturer's claim of significant results in one of the categories is not compelling.

Connections to Statistics and Expressions. The importance of randomized experimental design provides a connection with Statistics. Probability also has a more advanced connection with the Expression principle through Pascal's triangle and binomial expansions.

Core Skills · Students can and do:

1 Use methods of systematic counting to compute probabilities.

2 Take probability into account when making decisions and solving problems.

3 Compute theoretical probabilities and compare them to empirical results.

Include one- and two-stage investigations involving simple events and their complements, compound events involving dependent and independent simple events. Include using data from simulations carried out with technology to estimate probabilities.

4 Identify and explain common misconceptions regarding probability.

Include misconceptions about long-run versus. short-run behavior (the law of large numbers) and the "high exposure fallacy" (e.g., more media coverage suggests increased probability that an event will occur, which fails to account for the fact that media covers mostly unusual events).

5 Compute probabilities from a two-way table comparing two events.

Include reading conditional probabilities from two-way tables; do not emphasize fluency with the related formulas.

Statistics

Core Concepts · Students understand that:

A Statistics quantifies the uncertainty in claims based on data.

B Random sampling and assignment open the way for statistical methods.

C Visual displays and summary statistics condense the information in large data sets.

D A statistically significant result is one that is unlikely to be due to chance.

A Coherent Understanding of Statistics. We often base decisions or predictions on data. The decisions or predictions would be easy to make if the data always sent a clear signal, but the signal is usually obscured by noise. Statistical analysis aims to account for both the signal and the noise, allowing decisions to be as well informed as possible.

We gather, display, summarize, examine and interpret data to discover patterns. Data distributions can be described by a summary statistic measuring center, such as mean or median, and a summary statistic measuring spread, such as interquartile range or standard deviation. We can compare different distributions numerically using these statistics or visually using plots. Data are not just numbers, they are numbers that mean something in a context, and the meaning of a pattern in the data depends on the context. Which statistics to compare, and what the results of a comparison may mean, depend on the question to be investigated and the real-life actions to be taken.

We can use scatter plots or two-way tables to examine relationships between variables. Sometimes, if the scatter plot is approximately linear, we model the relationship with a trend line and summarize the strength and direction of the relationship with a correlation coefficient.

We use statistics to draw inferences about questions such as the effectiveness of a medical treatment or an investment strategy. There are two important uses of randomization in inference. First, collecting data from a random sample of a population of interest clears the way for inference about the whole population. Second, randomly assigning individuals to different treatments allows comparison of the their effectiveness. Randomness is the foundation for determining the statistical significance of a claim. A statistically significant difference is one that is unlikely to be due to chance; effects that are statistically significant may, nevertheless, be small and unimportant.

Sometimes we model a statistical relationship and use that model to show various possible outcomes. Technology makes it possible to simulate many possible outcomes in a short amount of time, allowing us to see what kind of variability to expect.

Connections to Probability, Expressions, and Numbers. Inferences rely on probability. Valid conclusions about a population depend on designed statistical studies using random sampling or assignment.

Core Skills · Students can and do:

1 Identify and formulate questions that can be addressed with data; collect and organize the data to respond to the question.

2 Use appropriate displays and summary statistics for data

Include univariate, bivariate, categorical and quantitative data. Include the thoughtful selection of measures of center and spread to summarize data.

3 Estimate population statistics using samples.

Focus on the mean of the sample, and exclude standard deviation.

4 Interpret data displays and summaries critically; draw conclusions and develop recommendations.

Include paying attention to the context of the data, interpolating or extrapolating judiciously and examining the effects of extreme values of the data on summary statistics of center and spread. Include data sets that follow a normal distribution.

5 Evaluate reports based on data.

Include looking for bias or flaws in way the data were gathered or presented, as well as unwarranted conclusions, such as claims that confuse correlation with causation.

Explanatory Problems

[Note: The Explanatory Problems are incomplete in this draft. Explanatory Problems will eventually appear alongside their corresponding standards when the standards move to a two-page format.]

The purpose of the Explanatory Problems is to explain certain Core Skills and exemplify the kinds of problems students should be able to do. This feature of the College and Career Readiness Standards has been modeled on the standards of Singapore, Japan, and other high-performing countries—as well as the standards of states like Massachusetts whose standards include such problems.

Explanatory Problems have been provided for those Core Skills in which difficult judgments must be made about the desired level of mathematical complexity. For Number and Modeling, no Explanatory Problems were judged necessary to further clarify the Core Skills.

Please note that the explanatory problems are specific cases and do not fully cover the content scope of their corresponding Core Skills. Also please note that these problems are not intended to be classroom activities. They are best thought of as parts of the standards statements to which they correspond.

Number

No Explanatory Problems intended

Expressions

1 See structure in expressions and manipulate simple expressions with procedural fluency.

Explanatory Problems (a)

Perform manipulations such as the following with procedural fluency:

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$
$(x+a)(x+b) = x^2 + (a+b)x + ab$

Expressions in (a) were taken from Japan COS, 2008

Explanatory Problems (b)

Simplify $\frac{(x^4)^2}{x^2}$

Simplify $\frac{12x}{y} - \frac{3xy}{y^2}$

Problems in (b) were taken from Hong Kong Secondary 3 Territory-Wide Assessment 2007

Explanatory Problem (c)

Expand fully $x\{1 - x(x+3)\}$

Problem (c) was taken from Singapore O Level January 2007 Exam

Additional Explanatory Problems to come

Expressions, continued

2 Write an expression to represent a quantity in a problem.

Explanatory Problems to come

Equations

2 Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality. Solve linear equations with procedural fluency. For quadratic equations, include solution by inspection, by factoring, or by using the quadratic formula.

Explanatory Problems to come

Functions

2 Describe the qualitative behavior of common types of functions using expressions, graphs and tables. Use graphs and tables to identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; and periodicity. Explore the effects of parameter changes (including shifts and stretches) on the graphs of these functions using technology. Include linear, power, quadratic, polynomial, simple rational, exponential, logarithmic, trigonometric, absolute value and step functions.

Explanatory Problems to come

3 Analyze functions using symbolic manipulation. Include slope-intercept and point-slope form of linear functions; factored form to find horizontal intercepts; vertex form of quadratic functions to find maximums and minimums; and manipulations as described under Expressions.

Explanatory Problems to come

4 Use the families of linear and exponential functions to solve problems. For linear functions $f(x) = mx + b$, understand b as the intercept or initial value and m as the slope or rate of change. For exponential functions $f(x) = a \cdot b^x$, understand a as the intercept or initial value and b as the growth factor.

Explanatory Problems to come

Quantity

4 Use units as a way to understand problems and to guide the solution of multi-step problems. Include examples such as acceleration; currency conversions; people-hours; social science measures, such as deaths per 100,000; and general rate, such as points per game.

Explanatory Problems to come

Modeling

No Explanatory Problems intended

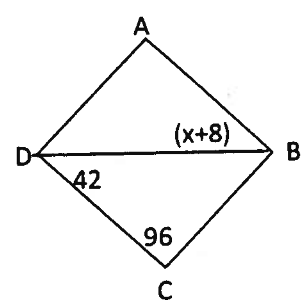
Shape

1 Use geometric properties to solve multi-step problems involving shapes. Include: measures of angles of a triangle sum to 180°; measures of vertical, alternate interior and corresponding angles are equal; measures of supplemental angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are equidistant from the segment's endpoints; and the radius of a circle is perpendicular to the tangent at the point of intersection of the circle and radius.

Explanatory Problem

This problem was taken from Hong Kong Secondary 3 Territory-Wide Assessment 2007

ABCD is a rhombus. Find x. Angle measurements are in degrees.



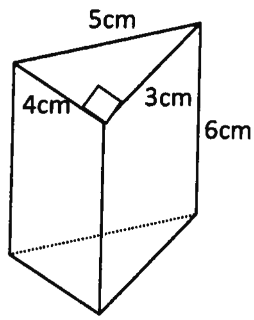
Shape, continued

3 **Solve problems involving measurements.** Include measurement (length, angle measure, area, surface area, and volume) of a variety of figures and shapes in two- and three-dimensions. Compute measurements using formulas and by decomposing complex shapes into simpler ones.

Explanatory Problem

This problem was taken from Hong Kong Secondary 3 Territory-Wide Assessment 2007

The figure shows a solid prism. Its base is a right-angled triangle. Find its surface area.



6 **Apply properties of right triangles and right triangle trigonometry to solve problems.** Include applying sine, cosine and tangent to determine lengths and angle measures of a right triangle, the Pythagorean theorem and properties of special right triangles. Use right triangles and their properties to solve real-world problems. Limit angle measures to degrees.

Explanatory Problem to come

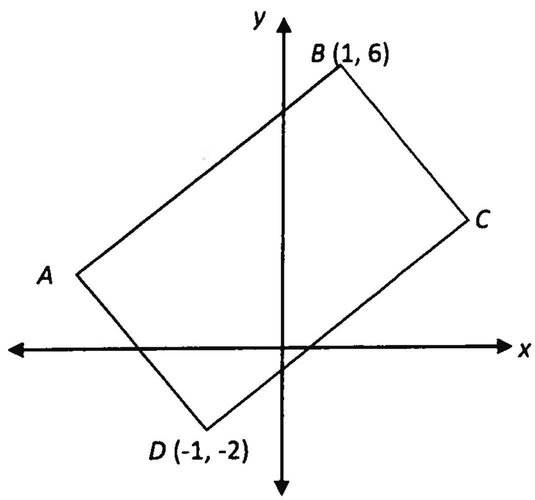
Coordinates

3 **Use geometric properties to solve multi-step problems involving shapes.** Include: measures of angles of a triangle sum to 180°; measures of vertical, alternate interior and corresponding angles are equal; measures of supplemental angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are equidistant from the segment's endpoints; and the radius of a circle is perpendicular to the tangent at the point of intersection of the circle and radius.

Explanatory Problem

This problem was taken from Hong Kong Secondary 3 Territory-Wide Assessment 2007

The figure below shows a rectangle ABCD. Find the length of the diagonal BD of the rectangle.



Probability

1 Use methods of systematic counting to compute probabilities.

Explanatory Problems to come

2 Take probability into account when making decisions and solving problems.

Explanatory Problems to come

Statistics

2 Use appropriate displays and summary statistics for data. Include univariate, bivariate, categorical and quantitative data. Include the thoughtful selection of measures of center and spread to summarize data.

Explanatory Problems to come

5 Evaluate reports based on data. Include looking for bias or flaws in way the data were gathered or presented, as well as unwarranted conclusions, such as claims that confuse correlation with causation.

Explanatory Problems to come

How Evidence Informed Decisions in Drafting the Standards

The Common Core Standards initiative builds on a generation of standards efforts led by states and national organizations. On behalf of the states, we have taken a step toward the next generation of standards that are aligned to college- and career-ready expectations and are internationally benchmarked. These standards are grounded in evidence from many sources that shows that the next generation of standards in mathematics must be focused on deeper, more thorough understanding of more fundamental mathematical ideas and higher mastery of these fewer, more useful skills.

The evidence that supports this new direction comes from a variety of sources. International comparisons show that high performing countries focus on fewer topics and that the U.S. curriculum is "a mile wide and an inch deep." Surveys of college faculty show the need to shift away from high school courses that merely survey advanced topics, toward courses that concentrate on developing an understanding and mastery of ideas and skills that are at the core of advanced mathematics. Reviews of data on student performance show the large majority of U.S. students are not mastering the mile wide list of topics that teachers cover.

The evidence tells us that in high performing countries like Singapore, the gap between what is taught and what is learned is relatively smaller than in Malaysia or the U.S. states. Malaysia's standards are higher than Singapore's, but their performance is much lower. One could interpret the narrower gap in Singapore as evidence that they actually use their standards to manage instruction; that is, Singapore's standards were set within the reach of hard work for their system and their population. Singapore's Ministry of Education flags its webpage with the motto, "Teach Less, Learn More." We accepted the challenge of writing standards that could work that way for U.S. teachers and students: By providing focus and coherence, we could enable more learning to take place at all levels.

However, a set of standards cannot be simplistically "derived" from any body of evidence. It is more accurate to say that we used evidence to inform our decisions. A few examples will illustrate how this was done.

For example, systems of linear equations were included by all states, yet students perform surprisingly poorly on this topic when assessed by ACT. We determined that systems of linear equations have high coherence value, mathematically; that this topic is included by all high performing nations; and that it has moderately high value to college faculty. Result: We included it in our standards.

A different and more complex pattern of evidence appeared with families of functions. Again, we found that students performed poorly on problems related to many advanced functions (trigonometric, logarithmic, quadratic, exponential, and so on). Again, we found that states included them, even though college faculty rated them lower in value. High performing countries included this material, but with different degrees of demand. We decided that we had to carve a careful line through these topics so that limited teaching resources could focus where it was most important. We decided that students should develop deep understanding and mastery of linear and simple exponential functions. They should also have familiarity (so to speak) with other families of functions, and apply their algebraic, modeling and problem solving skills to them—but not develop in-depth mastery and understanding. Thus we defined two distinct levels of attention and identified which families of functions got which level of attention.

Why were exponential functions selected in this case, instead of (say) quadratic functions? What tipped the balance was the high coherence value of exponential functions in supporting modeling and their wide utility in work and life. Quadratic functions were also judged to have received enough attention under Equations.

These examples indicate the kind of reasoning, informed by evidence, that it takes to design standards aligned to the demands of college and career readiness in a global economy. We considered inclusion in international standards, requirements of college and the workplace, surveys of college faculty and the business community, and other sources of evidence. As we navigated these sometimes conflicting signals, we always remained aware of the finiteness of instructional resources and the need for deep mathematical coherence in the standards.

In the pages that follow, the work group has identified a number of sources that played a role in the deliberations described above and more generally throughout the process to inform our decisions.

Sample of Works Consulted

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D. *Foundations for Success: Final Report of the National Mathematics Advisory Panel* (NMAP), (2008).

E. *Guidelines for Assessment and Instruction in Statistics Education (GAISE) project* http://www.amstat.org/education/gaise/.

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I. *The Opportunity Equation: Transforming Mathematics and Science Education for Citizenship and the Global Economy*. The Carnegie Corporation of New York and the Institute for Advanced Study, 2009. Online: http://www.opportunityequation.org/

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E. *Advanced Placement Calculus, Statistics and Computer Science Course Descriptions. May 2009, May 2010.* College Board, (2008).

F. *Aligning Postsecondary Expectations and High School Practice: The Gap Defined* (Policy Implications of the ACT National Curriculum Survey Results 2005-2006). Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/NCSPolicyBrief.pdf

G. *Condition of Education, 2004: Indicator 30, Top 30 Postsecondary Courses, U.S. Department of Education,* (2004).

H. *Condition of Education, 2007: High School Course-Taking.* U.S. Department of Education, (2007).

I. Conley, D.T. (2008). *Knowledge and Skills for University Success*

J. Conley, D.T. (2007). *Toward a More Comprehensive Conception of College Readiness.*

K. *Crisis at the Core: Preparing All Students for College and Work.* ACT, Last retrieved July 14, 2009, from www.act.org/research/policymakers/pdf/crisis_report.pdf

L. Achieve, Inc., Florida Postsecondary Survey, 2008.

M. Golfin, Peggy, et. al. CNA Corporation. (2005). *Strengthening Mathematics at the Postsecondary Level: Literature Review and Analysis.*

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X. Achieve, Inc., Virginia Postsecondary Survey, 2004.

III. Career Readiness

A. ACT Job Skill Comparison Charts

B. Achieve's Mathematics at Work, 2008. http://www.achieve.org/mathatwork

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IV. International Documents

STANDARDS [High performing countries/countries of interest]

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Department for Educational Development: Core Curriculum. (Grades 9-10 and 11-12) Retrieved June 24, 2009 from http://www.ond.vlaanderen.be/dvo/english/corecurriculum/corecurriculum.htm

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- Curriculum Guidelines for the Elective Subject Math (I) in Senior High Schools, as submitted to the APEC consortium. (Grades 12) Last retrieved July 13, 2009, from http://www.apecknowledgebank.org/file.aspx?id=1901

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Qualifications and Curriculum Authority. (2007). *Programme of Study for Key Stage 4, National Curriculum, 2007*. (Grades 10-11)

F. Finland

Finnish National Board of Education. (2003). *National Core Curriculum For Upper Secondary Schools 2003*. (Grades 10-12)

G. Hong Kong

Learning Objectives for Key Stage 4. (Grades 10-11)

H. India

- *The National Curriculum Framework*. (2005). (Grades 9-12). National Council of Educational Research and Training. Last retrieved July 13, 2009, from http://ncert.nic.in/html/pdf/schoolcurriculum/framework05/prelims.pdf
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I. Ireland

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L. Singapore

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ASSESSMENTS [Key international assessments with global reach]

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B. Trends in International Mathematics and Science Study (TIMSS), (2007).

C. International Baccalaureate, Mathematics Standard Level, (2006).

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V. State Documents

A. California State Standards

B. Florida State Standards

C. Georgia State Standards

D. Massachusetts State Standards

E. Mathematics Curriculum Framework Revision Panel [Revised Progress Report]. Massachusetts Department of Elementary and Secondary Education, (2009). Last retrieved July 15, 2009, from http://www.doe.mass.edu/boe/docs/0509/item5_report.pdf.

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Common Core State Mathematics Standards for College and Work Readiness

Examples

Review Draft 1 – June 11, 2009

This document is not complete; it is intended to give a first-order picture of this portion of the project.

Early reviewers provided feedback on this document in June. Although their comments have not yet been implemented, we are resubmitting the original document to the larger group for additional comments. This document has not changed since June 11.

Example Performance Tasks in Mathematics*

The tasks in this document exemplify performance targets for college ready mathematics. Representing a variety of important task types, they have been developed for use in instructional contexts such as homework, classroom work, and assessments embedded in instruction.

Performance tasks in mathematics do (and should) vary along a number of dimensions:

- The relative balance of procedural fluency, conceptual understanding, and strategic capacity;
- The duration of the task, ranging from a minute or less, to a day or more;
- The topic coverage, from narrow content focus to end of course, integrated tasks;
- The relative balance of reasoning, communication, and making connections;
- The purposes of diagnostic/formative assessment vs. summative assessment, including assessment for accountability purposes
- The circumstances of the performance, whether individual or group, on-demand or as part of coursework.

For simplicity, the tasks here are grouped into the following four categories roughly ordered by the length of the chains of reasoning they demand and, linked to this, the balance between the strategic and the technical skills they demand or promote. The system is admittedly heuristic, and the classification of any given task is open to debate. In subsequent drafts, explicit references will be included to Principles and Performances being exemplified.

Target Tasks

These exemplify "realistic" situations, where "realism" includes not only "real-life" situations of describing phenomena, but also situations of conjecture and proof exemplifying "authentic" mathematics. Target tasks may be directive ("Design…", "Prove that…") or they may be open ("Find the properties of…", "What will happen if…"). They require students to integrate strategic, tactical and technical skills through connections within mathematics and to the problem context. Some target tasks allow good responses in only 10-20 minutes, though many can stimulate hours of valuable investigation.

Substantial Tasks

These are versions of target tasks that provide enough scaffolding to help students grasp the core problem by providing specific questions, while encouraging others. They can cover all aspects of performance but with less emphasis on question-posing and strategy.

Structured Tasks

Heavily-scaffolded versions of target tasks that provide a ramp of challenge through subtasks of increasing complexity and/or generality and/or abstraction, linked by the problem context. The length of the longest subtask is often just a few minutes.

Exercises

Short items (or groups of them), typically taking 2 minutes each or less, that assess or build specific concepts and technical skills.

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* Adapted in part from materials originally developed by the Shell Centre

Example Performance Tasks

Exercises

- "Exponential Expressions"
- "Quadratic Equations"
- "Classifying Quadrilaterals"
- "Equation of line"
- "Function Inputs vs. Values"
- "Ordering Expressions"
- "Shaded Triangle"
- "Left or Right"
- "Gamers"

Structured Tasks

- "How Much Rice?"
- "Consecutive Sums"
- "Irrational Triangles"
- "Hot Under the Collar"
- "Journey Time"
- "Sugar Prices"
- "Two Solutions"
- "Counting Trees"
- "Security Camera"

Substantial Tasks

- "The Spaghetti Measure"
- "Ponzi Scheme"
- "Mailing Cost Problem"
- "Rhombus Construction"

Target Tasks

- "Wheel Chair Ramp"
- "Maximum Volume"
- "Consecutive Sums" (less structured version)
- "How Long is a Roll of Paper?"
- "Two-Second Rule"

Exercises

Exponential Expressions

From McCallum, Connally, Hughes-Hallet et al., *Algebra: Form and Function*, 2010

Although *Algebra: Form and Function* is a college level textbook, the exercises shown here are also appropriate as high school exercises building readiness for college algebra.

Prices are increasing at 5% per year. What is wrong with the statements in Problems 16-24? Correct the formula in the statement.

16. A $6 item costs $\$(6 \cdot 1.05)^7$ in 7 years' time.

17. A $3 item costs $\$3(0.05)^{10}$ in ten years' time.

18. The percent increase in prices over a 25-year period is $(1.05)^{25}$.

19. If time t is measured in months, then the price of a $100 item at the end of one year is $\$100(1.05)^{12t}$.

20. If the rate at which prices increase is doubled, then the prices of a $20 object in 7 years' time is $\$20(2.10)^7$.

21. If time t is measured in decades (10 years), then the price of a $45 item in t decades is $\$45(1.05)^{0.1t}$.

22. Prices change by $10 \cdot 5\% = 50\%$ over a decade.

23. Prices change by $(5/12)\%$ in one month.

24. A $250 million town budget is trimmed by 1% but then increases with inflation as prices go up. Ten years later, the budget is $\$250(1.04)^{10}$ million.

Quadratic Equations

Adapted from McCallum, Connally, Hughes-Hallet et al., *Algebra: Form and Function*, 2010

Although *Algebra: Form and Function* is a college level textbook, the exercises shown here are also appropriate as high school exercises building readiness for college algebra.

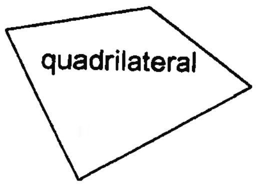
Preferably without solving them all the way, say whether the equations in Problems 61-68 have two solutions, one solution, or no solution. Give a reason for your answer.

- $3(x-3)(x+2)=0$
- $(x+5)(x+5)=-10$
- $(x-3)^2=0$
- $-2(x-1)^2+7=5$
- $(x-2)(x-2)=0$
- $(x+2)^2=17$
- $3(x+2)^2+5=1$
- $2(x-3)^2+10=10$

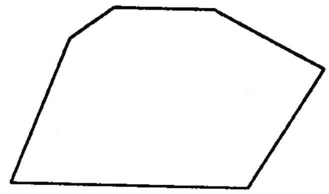
Classifying Quadrilaterals

This task is about classifying quadrilaterals. You are asked to classify quadrilaterals according to the number of right angles and the number of pairs of parallel sides.

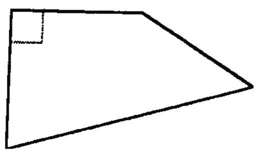
Quadrilateral is the "family" name that is given to closed shapes with four sides.



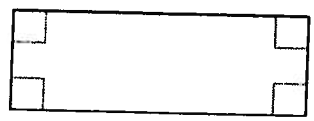
This closed shape has four sides; it is a quadrilateral.



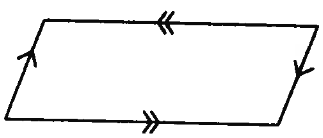
This is not a quadrilateral.



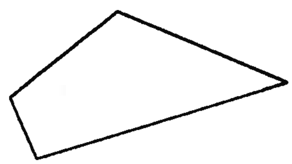
This quadrilateral has one right angle.



This quadrilateral has four right angles.



This quadrilateral has two pairs of parallel sides.



This one has no pairs of parallel sides.

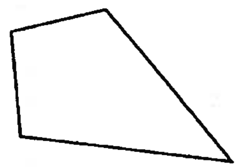
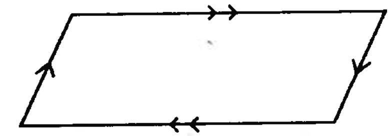
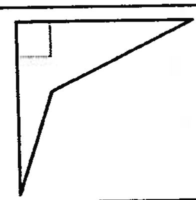
In the boxes provided, sketch a quadrilateral that has both of the properties associated with each box, if possible. Sketch it into the appropriate box. Label the right angles and sides that are parallel.

If it is impossible to fill a box, then justify why you cannot do so.

Some of the boxes have been filled in for you.

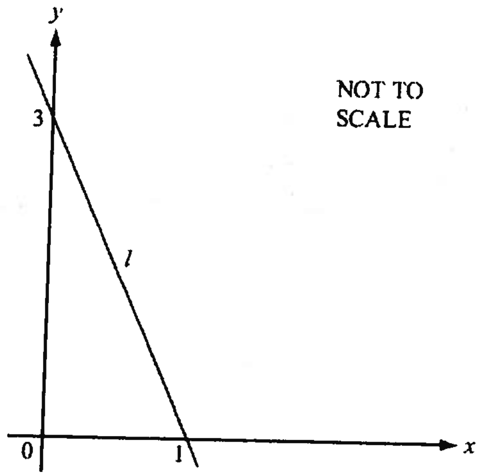
continued on next page

Number of parallel sides (exactly)

Number of right angles (exactly)	0	1	2
0			
1			
2			
3			
4			

Equation of line

12



A straight line, l, crosses the x-axis at (1, 0) and the y-axis at (0, 3).

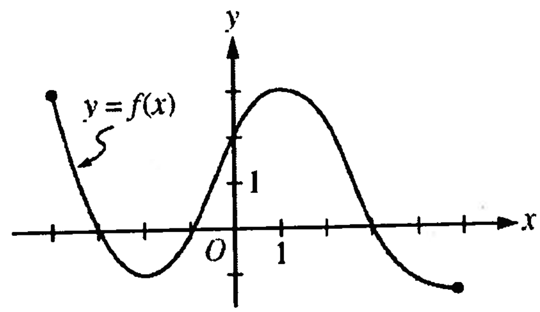
(a) Find the gradient of the line l.

Answer(a) .... [1]

(b) Write down the equation of the line l, in the form $y = mx + c$.

Answer(b) $y =$ .... [2]

Function Values vs. Inputs



The function f, defined for $-4 <= x <= 5$, is graphed in the xy-plane above. For how many values of x does $f(x) = 2$?

Ordering Expressions

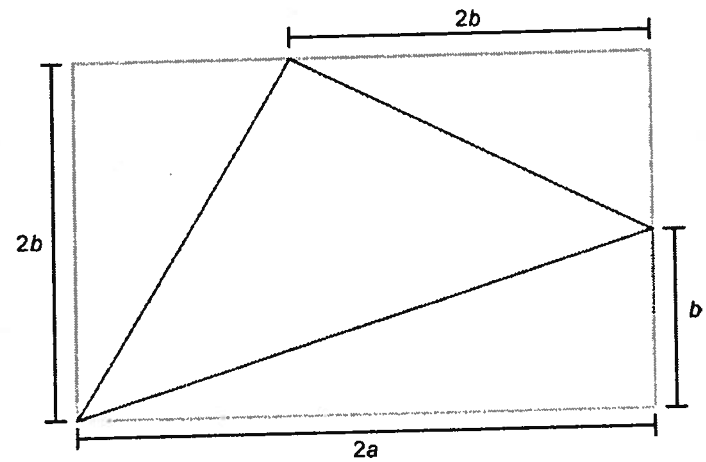
$$j = x^2 - 0.49$$
$$k = (x - 0.49)^2$$
$$m = x^3 - 0.49$$

If x is a negative integer, what is the ordering of j, k, and m from least to greatest?

Shaded Triangle

The diagram below shows a rectangle, with a shaded triangle inside.

Create an expression for the area of the shaded triangle.



Left or Right?–Using Linear Functions

The graphs for all these functions are straight lines:

$y = -2x - 5$	$y = -x - 5$	$y = x - 5$	$y = 2x - 5$
$y = -2x - 2$	$y = -x - 2$	$y = x - 2$	$y = 2x - 2$
$y = -2x + 1$	$y = -x + 1$	$y = x + 1$	$y = 2x + 1$
$y = -2x + 4$	$y = -x + 4$	$y = x + 4$	$y = 2x + 4$

1. Find pairs of functions from this list whose graphs do not intersect.

2. Find pairs of functions from this list whose graphs intersect to the left of the y-axis.

3. Find pairs of functions from this list whose graphs intersect to the right of the y-axis.

4. Find pairs of functions from this list whose graphs intersect on the y-axis.

5. Explain how you might predict, without actually graphing,

a. whether the graphs will meet;

b. whether they will meet on the y-axis;

c. whether they will meet on the left or right of the y-axis.

Gamers

Bobby and Esmeralda, two avid video game players, found a list of the top 100 video games of all time. They discovered that the list had many titles they did not know. The students created a two-way frequency table (also known as a contingency table) displaying the number of games on the list that they had played.

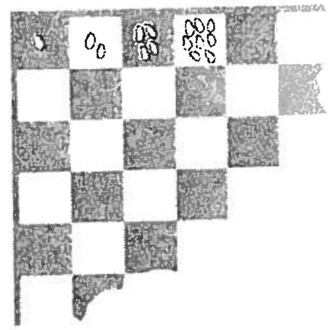
	Esmeralda played	**Esmeralda did not play**	**Total**
Bobby played	6	14	20
Bobby did not play	4	76	80
	10	90	100

Determine the probability that if a game was selected at random from the list—

1. Both Esmeralda and Bobby had played the game.
2. Esmeralda had played the game.
3. Only Bobby had played the game.
4. Exactly one of the students had played the game.
5. Bobby had not played the game.

Structured Tasks

How Much Rice?



According to legend, the game of chess was invented for a king by one of his servants. The king told his servant that he could have whatever he asked for as a reward.

The servant asked for one grain of rice on the first square of the chessboard, two grains on the second square, four grains on the third square, and so on, each square having twice as many grains as the square before.

At first the king thought that the servant's request was reasonable.

Soon, however, he realized just how much rice the servant had asked for.

How much rice did the servant ask for? What is a good estimate for the weight of this amount of rice?

What is a good estimate for the volume of this amount of rice?

What proportion of the total amount of rice is contributed by the 64^{th} square of the chessboard?

CONSECUTIVE SUMS 2

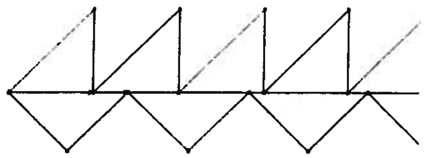
The number 15 can be written as the sum of consecutive whole numbers in three different ways:

$$15 = 7 + 8$$
$$15 = 1 + 2 + 3 + 4 + 5$$
$$15 = 4 + 5 + 6$$

1. Show that any odd number can be written as a consecutive sum
2. What properties does the sum of three consecutive numbers have?
3. What properties does the sum of four consecutive numbers have?
4. Explain any other properties of consecutive sums you can find.
5. What numbers cannot be written as consecutive sums, and why?

IRRATIONAL TRIANGLES

Consider the following arrangement of congruent 45°-45°-90° triangles in the plane. If the pattern is continued to the right, when if ever, will it happen that two triangles end at the same point?



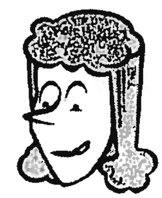
(continue the sequence further)

HOT UNDER THE COLLAR

John and Anne are discussing how they change temperatures in degrees Celsius into degrees Fahrenheit.

John

Anne

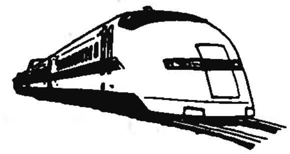


1. If the temperature is 20° C, what is this in Fahrenheit?
   How far out will Anne be, if she uses her method?

2. For what temperatures does Anne's method give an answer that is too high?

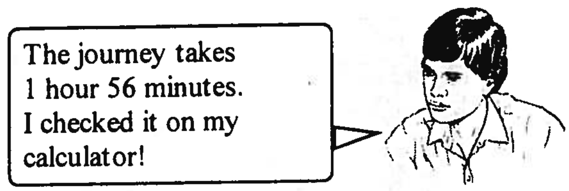
3. Write algebra expressions for both rules and graph both rules on a single set of axes.

Journey time



Mike's train leaves at 7.51 am
It arrives in New York at 9.07 am
Mike works out his journey time like this:

$$\begin{array}{r} {}^{8}\not{9}.{}^{1}07 \; - \\ 7.51 \\ \hline 1.56 \\ \hline \end{array}$$



Is Mike correct? ....

If not, find the correct journey time and explain what Mike did wrong.

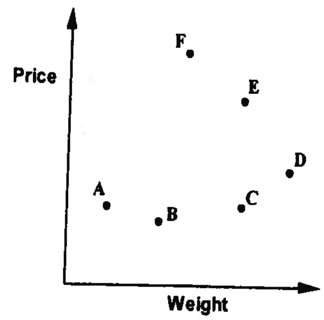
....

....

....

Sugar Prices

Each point on this graph represent a bag of sugar.



(a) Which point shows the heaviest bag? ....

(b) Which point shows the cheapest bag? ....

(c) Which points show bags with the same weight? ....

(d) Which points show bags with the same price? ....

(e) Which of F or C gives the best value for money? ....
How can you tell?

....

....

(f) Which of B or C gives the best value for money? ....
How can you tell?

....

....

(g) Which two bags give the same value for money? ....
How can you tell?

....

....

Two Solutions

This problem gives you the chance to:
- find solutions to equations and inequalities

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ __________ __________

b. $x^2 = x$ __________ __________

c. $x^2 < x$ __________ __________

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ __________ __________

e. $1776x + 1066 \geq 365$ __________ __________

f. $x^2 > x^3$ __________ __________

g. $|x| > x$ __________ __________

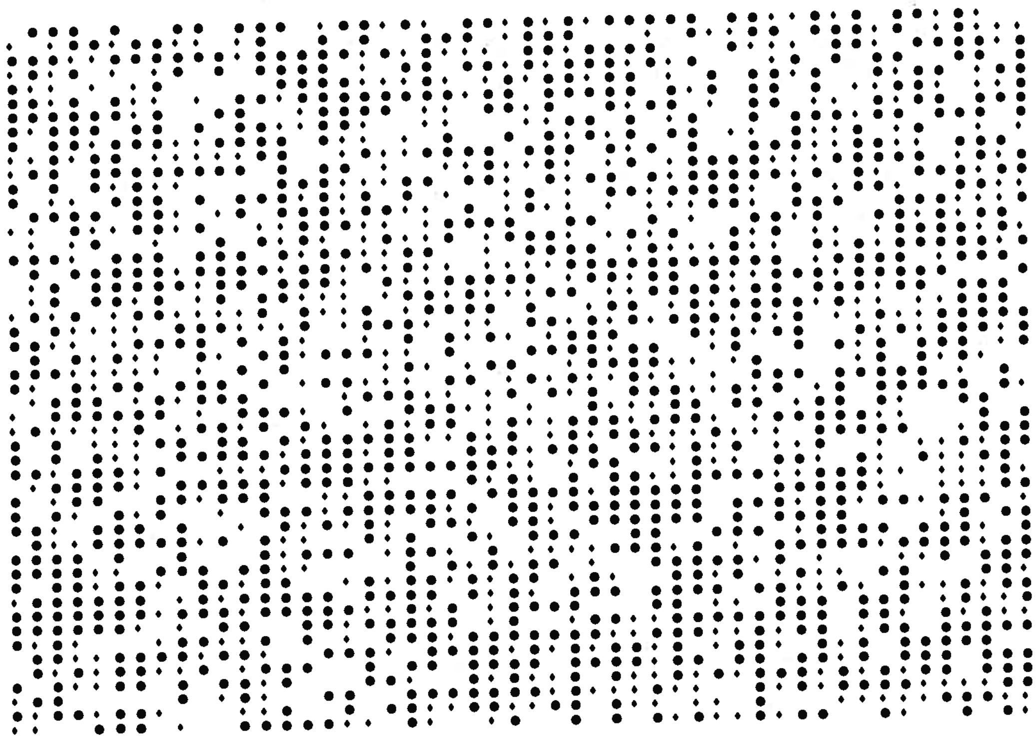
2. Some of the equations and inequalities on the page opposite have exactly two solutions; others have more than two solutions.

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

c. Write down two equations or inequalities that have an infinite number of solutions.

COUNTING TREES



This diagram shows some trees in a park.

The circles ● show old trees and the triangles ♦ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type? Explain your method fully.

2. On your worksheet, use your method to estimate:

   (a) The number of old trees in the park

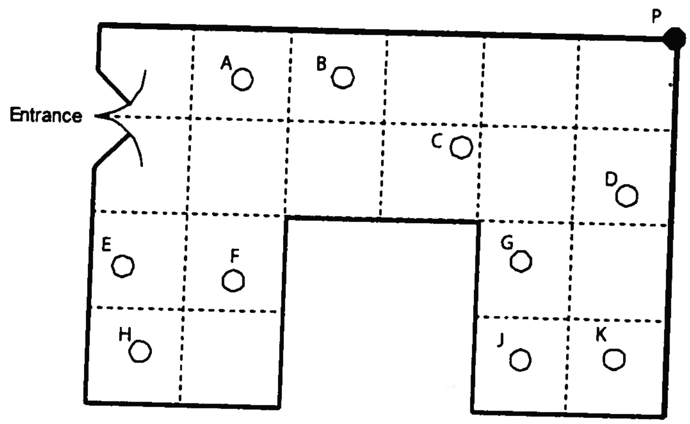
   (b) The number of young trees in the park

   (c) The percentage of trees in the park that are old.

Security camera

A shop owner wants to prevent shoplifting.
He decides to install a security camera on the ceiling of his shop.
The camera can turn right round through 360°.
The shop owner places the camera at point P, in the corner of the shop.

Plan view of the shop



1. The plan shows ten people are standing in the shop.
   These are labelled A, B, C, D, E, F, G, H, J, K.
   Which people cannot be seen by the camera at P ?

2. The shopkeeper says that "15% of the shop is hidden from the camera"
   Show clearly that he is right.

3. (a) Show the best place for the camera, so that the it can see as much of the shop as possible.

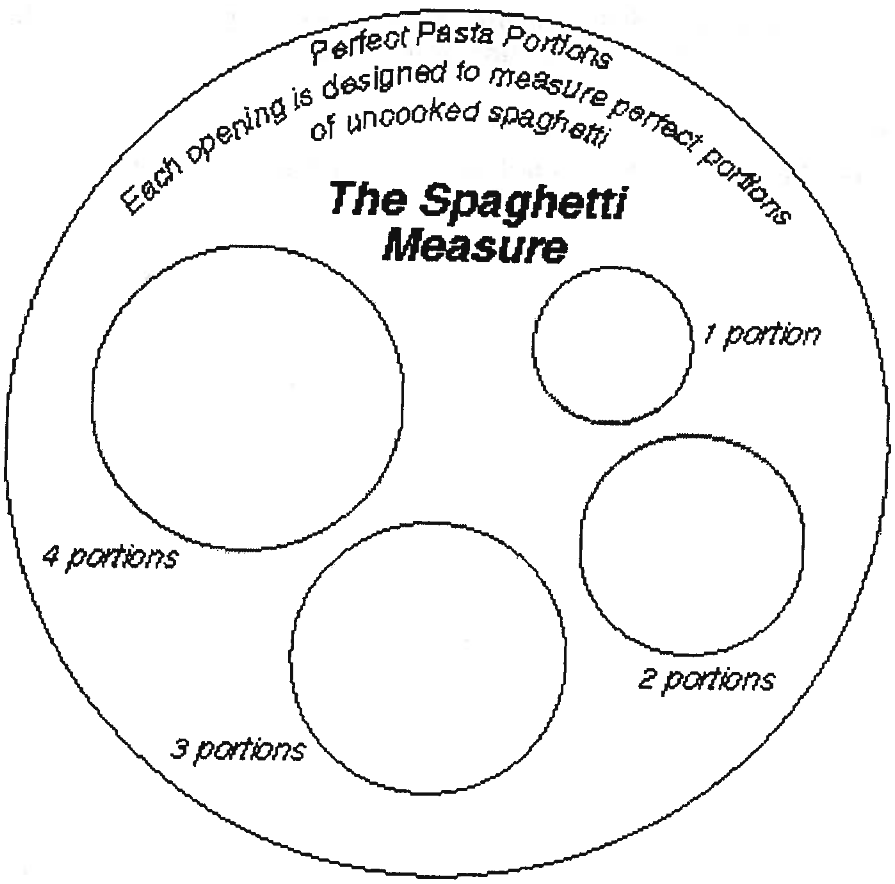
   (b) Explain how you know that this is the best place for the camera.

Substantial Tasks

This spaghetti measure is designed for use in the kitchen to figure out the amount of uncooked spaghetti needed for one, two, three, or four cooked portions.

For example, if you were measuring the amount of spaghetti for three portions, you would place as much spaghetti as possible in the hole labeled "3 portions."

The spaghetti measure is drawn full size.



An investigation

In this spaghetti measure, which quantities are proportional to which other quantities?

Criteria for success on this task

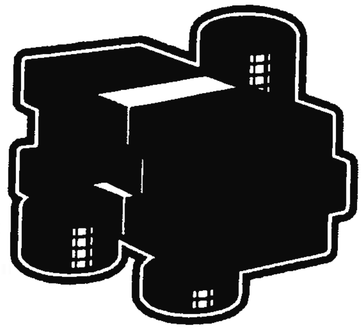
Make sure that your spaghetti-measure investigation uses graphs and formulae to answer these questions:

1. **In the spaghetti measure, is “portion size” proportional to the diameter of an opening? Explain how you know.**
2. **In the spaghetti measure, is “portion size” proportional to the circumference of an opening? Explain how you know.**
3. **In the spaghetti measure, is “portion size” proportional to the area of an opening? Explain how you know.**

Possible task extension

Specify the size of an opening that would be needed to measure 5 portions of spaghetti.

"PONZI" PYRAMID SCHEMES



Max has received this email. It describes a scheme for making money.

From: A Crook
Date: Thursday 15th January 2009
To: B Careful
Subject: Get rich quick!

Dear friend,

Do you want to get rich quick? Just follow the instructions carefully below and you may never need to work again:

1. At the bottom of this email there are 8 names and addresses. Send $5 to the name at the top of this list.
2. Delete that name and add your own name and address at the bottom of the list.
3. Send this email to 5 new friends.

1. If that process goes as planned, how much money could be sent to Max?

2. What could possibly go wrong?
   Explain your answer clearly.

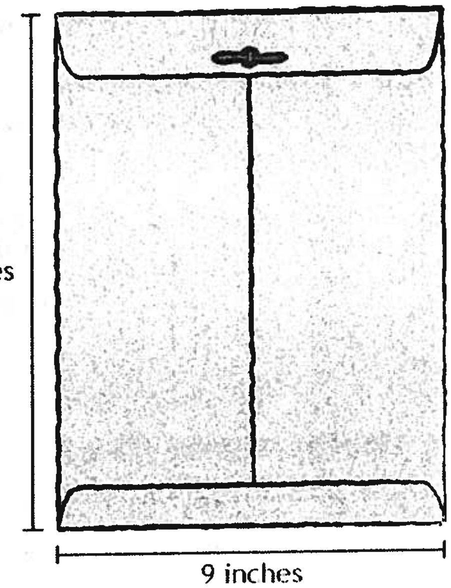
3. Why do they make Ponzi schemes like this illegal?

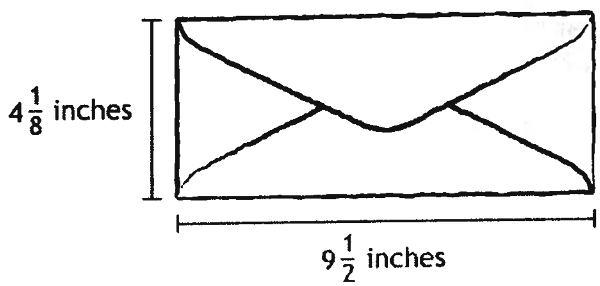
Mailing cost problem!

Linda does volunteer work and often has to mail out documents.

The number of sheets of paper in the envelope might be anywhere from 1 to 25.

Linda can use one of two types of envelope for each document:





Next to her desk Linda has pinned the following information:

LETTERS

Weight Not Over	Price
1 ounce	$0.44
2 ounces	$0.61
3 ounces	$0.78
3.5 ounces	$0.95

Size limits:
- Rectangular; length is the dimension parallel to the address.
- At least $3\frac{1}{2}$ inches high by 5 inches long by 0.007 inch thick
- No more than $6\frac{1}{8}$ inches high by $11\frac{1}{2}$ inches long by $\frac{1}{4}$ inch thick
- Up to 3.5 ounces

LARGE ENVELOPES

Weight Not Over	Price
1 ounce	$0.88
2 ounces	$1.05
3 ounces	$1.22
4 ounces	$1.39
5 ounces	$1.56
6 ounces	$1.73
7 ounces	$1.90
8 ounces	$2.07
9 ounces	$2.24
10 ounces	$2.41
11 ounces	$2.58
12 ounces	$2.75
13 ounces	$2.92

Size limits:
- Rectangular
- No more than 12 inches high by 15 inches long by $\frac{3}{4}$ inch thick
- Must not be rigid and must be uniformly thick

NOTES

10 envelopes $4\frac{1}{8}$ inches x $9\frac{1}{2}$ inches weigh 3 ounces

10 envelopes 9 inches x 12 inches weigh 8 ounces

100 sheets of paper weigh 16 ounces

500 sheets of paper are 2 inches thick

Mailing cost problem!

Each time that Linda needs to find the exact cost of mailing an envelope, she has to weigh it. She would like to find another way that does not involve weighing.

Also, Linda would like to save as much money as possible by mailing each envelope in the most economical way.

Please help Linda in the following two ways:

Make a table that will help Linda find the cost of mailing each envelope.

Your table should be simple and easy to use, and it should help Linda spend as little as possible on postage.

TO DO

Make a graph of the information in your table.

Again, try to make it simple and easy to use.

TO DO

Rhombus Construction?

The diagonals of a rectangle divide it into four isosceles triangles.

Here is a way to find a rhombus (that has no right angles) with the same property. Let s be the side length of the rhombus. As the acute angle in a rhombus becomes smaller, the long diagonal of the rhombus gets larger. Reduce the acute angle of the rhombus until the long diagonal increases to length $2s$. Then draw the short diagonal of this rhombus. The diagonals of a rhombus (or any parallelogram) bisect each other, so the long diagonal has been divided into two segments of length s. Thus each of the four triangles formed by the two diagonals of the rhombus have two sides of length s. Hence each of the four triangles is isosceles.

Is this construction correct? If not, what is wrong with it?

Target Tasks

Your task is to design a ramp that will provide wheelchair access from the outside of the school to an exterior second-floor door.

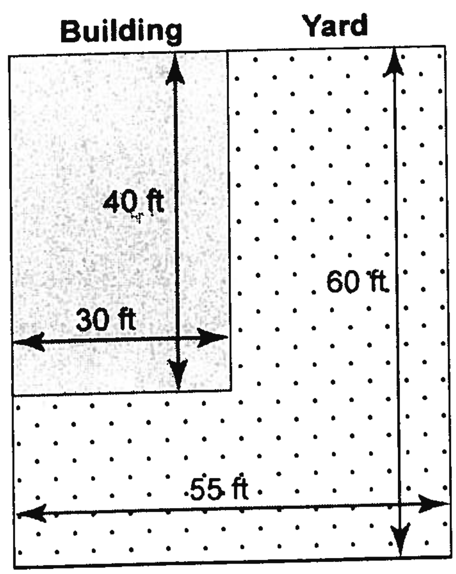
Your ramp must comply with the Americans with Disabilities Act (ADA). See Section 4.8 (Ramps) of the ADA Accessibility Guidelines For Buildings And Facilities, within the ADA Standards for Accessible Design, available at http://www.ada.gov/stdspdf.htm.

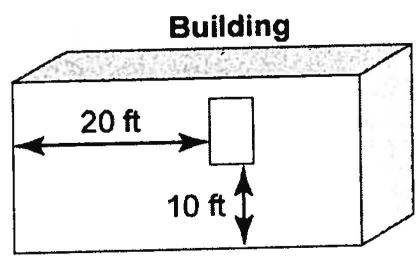
You can do one of two things:

You can use your own school building to locate a suitable door on the second floor and work within the actual constraints presented by the outside of your school and the ADA.

Or you can create a ramp to access the building described below.

A high school is doing renovations. A wheelchair ramp is needed to provide access to a door that is 10 feet above the ground.





(The diagrams are not drawn to scale.)

Imagine that you are the architect who is to design this ramp for the school. You must:

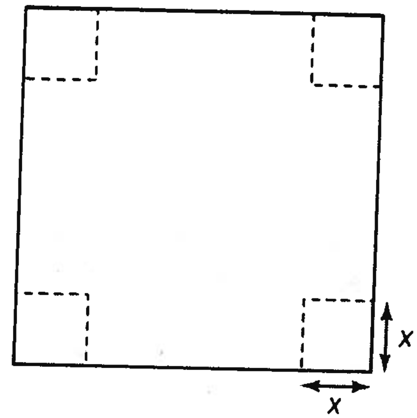
Draw a diagram to show how you created the access. Communicate clearly your design decisions. Include your calculations. Show how each specification of the ADA is met. Produce a scale model.

A company displays its candies in small, open-topped boxes that have no lid. To save paper, the company is trying to maximize the volume of its boxes.

You can make a box without a lid by cutting a square of side x centimeters from each corner of a larger square and folding the remaining paper.

To make the boxes, the company has available squares of construction paper that range in side length from 6 centimeters through 20 centimeters.

For any given size of construction paper, what side of square must you cut out from the corners in order to maximize the volume of the box?



Generalizing your solution:

Suppose you start with a square piece of construction paper of side S. What side of square must you cut from the corners of the construction paper in order to maximize the volume of the box?

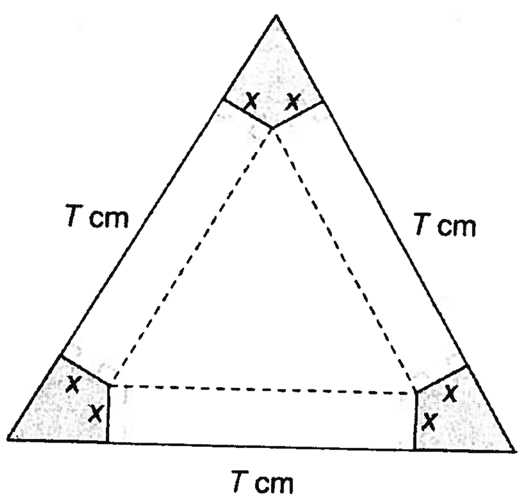
Your investigation must include models, tables, and graphs.

For a box with maximum volume constructed from a square of side S, what is the relationship between the area of the base and the total surface area of the sides?

Extending your solution:

You have a piece of construction paper that is in the shape of an equilateral triangle of side T centimeters. You want to make a box, without a lid, by cutting congruent quadrilaterals from the three corners as shown in the diagram below.

What value of x will make the volume of the box a maximum?



Do your generalizations hold when you change the shape of the base of the container? What happens if the base of your container is a circle?

CONSECUTIVE SUMS

The number 15 can be written as the sum of consecutive whole numbers in exactly three different ways:

$$15 = 7 + 8$$
$$15 = 1 + 2 + 3 + 4 + 5$$
$$15 = 4 + 5 + 6$$

The number 9 can be written as the sum of consecutive whole numbers in only two ways:

$$9 = 2 + 3 + 4$$
$$9 = 4 + 5$$

Now look at other numbers and find out all you can about writing them as sums of consecutive whole numbers.

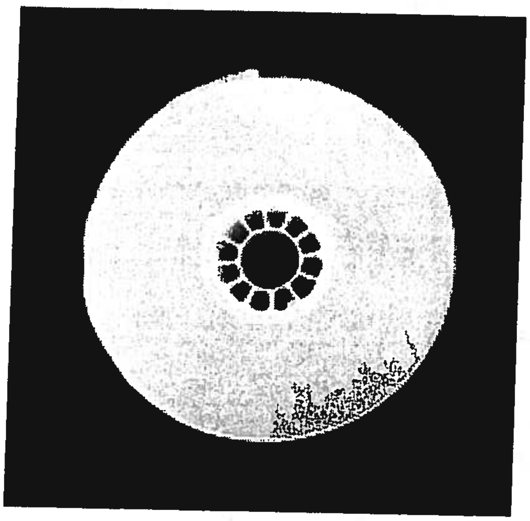
Write an account of your investigation. If you find any patterns in your results, then try to explain them fully.

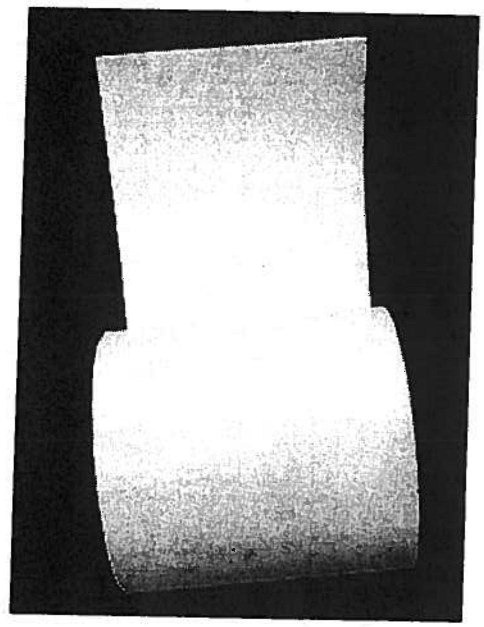
How long is a roll of paper?

Your job is to find a good approximation for the length of a roll of paper.[1]

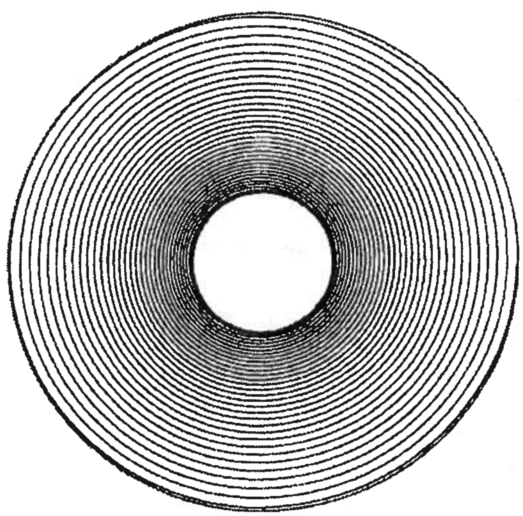
You must not attempt to solve the problem by unrolling and measuring the length of the paper.

You may use any roll. The photograph below shows two views of a roll that we used:





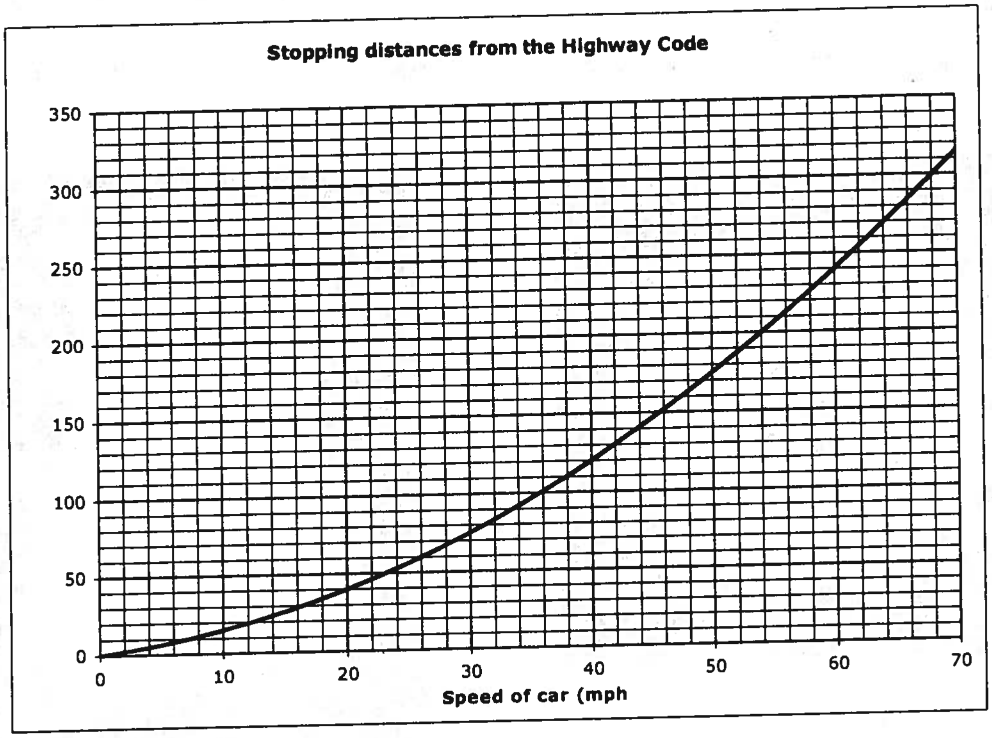
Also, here is a scale drawing of the roll shown in the pictures:



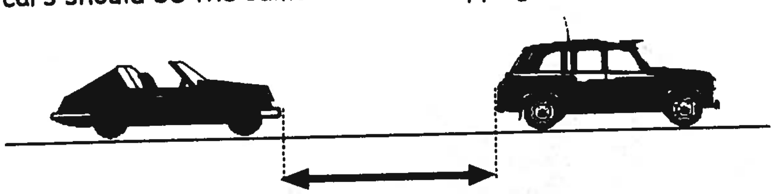
[1] This task was conceived by Dick Stanley.

TWO-SECOND RULE

The distance needed for a car to stop is called the *stopping distance*.
The graph below shows how the stopping distance depends on the speed of the car.



A Highway Code booklet says that, for safety, the smallest gap between two cars should be the same as their stopping distance:



Max cannot estimate distances very well, so he uses the following 'two-second rule' instead:
"The distance between two cars should be the distance the second car would travel in two seconds."

1. Is it 'safe' for Max to use the two-second rule when driving at 30 mph?
   (1 mile = 5280 feet).

2. When is the two-second rule 'unsafe'?
   Show your reasoning clearly.